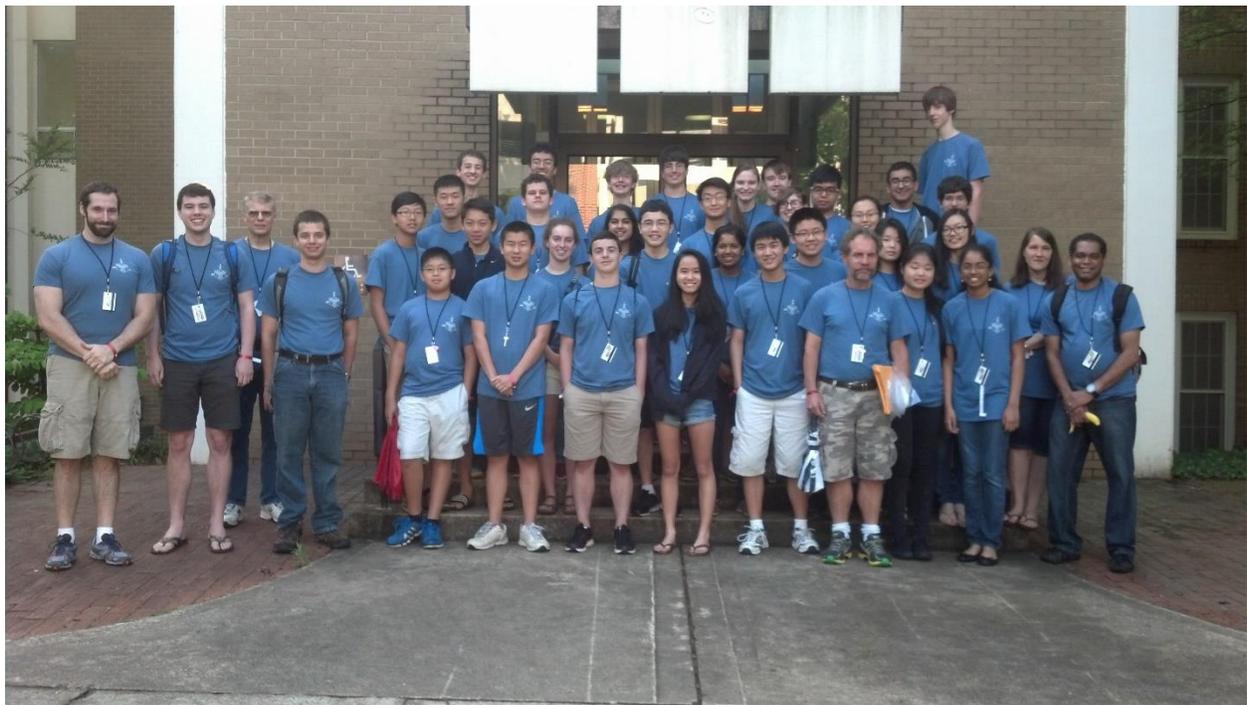


THE MATHMATE



*THE OFFICIAL JOURNAL OF THE
SOUTH CAROLINA COUNCIL OF TEACHERS OF MATHEMATICS*

Editorial Board

Lead Editor

GINA POORE DUNN, *Lander University*

SCCTM Officers

President

JENNIFER WILSON, *Anderson District 5*

President Elect

CHRISTI FRICKS, *Lakeside Middle School*

Past President

KAREN THRELKELD, *T.L. Hanna High School*

Vice President for Elementary

JOYCE MOON, *Cane Bay Elementary School*

Vice President for Middle

BERNARD FROST, *Fairforest Middle School*

Vice President for Secondary

AMANDA McKEE, *Johnsonville High School*

Vice President for Post Secondary

LEIGH HALTIWANGER, *Clemson University*

Vice President at Large

JERRY MARSHALL, *Tri-County Technical College*

Secretary

SUZANNE PETERSON, *Pelion Middle School*

Treasurer

TERRI DEW, *S²tem Centers SC*

NCTM Representative

SANDY POWERS, *College of Charleston*

Government Relations

MEGAN CHE, *Clemson*

SC Department of Education Representative

HARRIET PRITCHARD

Database/Membership Manager

BILL STEVENS

Mission Statement: The mission of THE MATHMATE is to feature articles about innovative mathematical classroom practices, important and timely educational issues, pedagogical methods, theoretical findings, significant mathematical ideas, and hands-on classroom activities and disseminate this information to students, educators and administrators.

THE MATHMATE, the official journal of the South Carolina Council of Teachers of Mathematics, is published online three times each year – January, May, and September.

Submission Requirements: Submissions for THE MATHMATE should be no more than 15 pages in length not counting cover page, abstract, references, tables, and figures. Submissions of more than 15 pages will be reviewed at the discretion of the editorial board. Submissions should conform to the style specified in the *Publications Manual of the American Psychological Association* (6th ed.). All submissions are to be emailed to scmathmate@gmail.com as attachments with a completed Submission Coversheet as page 1 and the article starting on page 2. [Click here to download THE MATHMATE Submission Coversheet.](#)

Submitted files must be saved as MSWord, RTF, or PDF files. Pictures and diagrams must be saved as separate files and appropriately labeled according to APA style. Copyright information will be sent once an article is reviewed but authors should not submit the same article to another publication while it is in review for THE MATHMATE.

Submission Deadlines: Submissions received by October 1 will be considered for the January issue, February 1 for the May issue, and June 1 for the September issue.

South Carolina Recertification Credit: According to the SC Department of Education Renewal Credit matrix, <http://ed.sc.gov/agency/se/Educator-Services/Licensure/documents/renewalmatrix2013.pdf>, the primary author of a refereed journal article can earn 60 renewal credits.

Subscriptions: Active members of SCCTM receive online subscriptions to THE MATHMATE as part of their memberships.

Correspondence: All correspondence regarding THE MATHMATE should be sent to scmathmate@gmail.com

Permission To Photocopy: Active members of SCCTM may distribute copies of articles published in THE MATHMATE individually and in limited quantities.

Current Peer Reviewers:

Chris Duncan, *Lander University*
Leigh Haltiwanger, *Clemson University*
Josie Ryan, *Lander University*
Amber Simpson, *Clemson University*

THE MATHMATE

Table of Contents

- 5 Message from the SCCTM President
- 6 Announcements
- 7 Guest Editorial
Marc Drews
- 9 An Implementation of the Tabor Rotation Method in an Elementary Math Methods Course – Artifacts, Student Feedback, and Instructor Impressions
Gary Bradley
- 17 Word Numbers Fortified by Universal Design for Learning
Diana Cheng and Nicole Horner
- 26 Challenging Students’ Conceptions of Proportional Reasoning
Leigh Haltiwanger and Amber Simpson
- 37 A Technology Makeover for the Glyph
Pamela D. Wash and Elizabeth L. Syracuse

Message from the SCCTM President

Dear Members,

Hello! Welcome to another school year! I hope your year is progressing along nicely wherever you are and with whatever you currently teach. With all of the Common Core and educational leadership changes that are forthcoming, one thing remains steadfast: our love of mathematics and the students of our state!

Speaking of fall and all it brings, I'd like to remind our members of a few important items. First, our fall conference is right around the corner! The SCCTM will host hundreds (maybe even thousands!) of educators in Myrtle Beach on November 6-7 at the Myrtle Beach Convention Center. Hopefully, you have made your arrangements to be there. We have excellent speakers, authors, and sessions in store for those in attendance. Not to mention, Myrtle Beach is a great area to visit and explore. Secondly, please remember to vote for our SCCTM Board candidates. The online voting will be open on October 1 and will run through November 5 at noon. Please be on the lookout for the link to vote for your favorite person. We have very worthy educators vying for positions on our board. Additionally, if you are a registered voter, please do so this November. Teachers have a voice that should be heard. Elect who you think will lead us in the right direction with our children's best interest at heart. Finally, as a member of the best mathematics organization around, let's celebrate who we are and all that we do! If you have an amazing lesson or story to share, please do so. We gladly accept articles for the newsletter and the Mathmate. If you or someone in your district is doing something awesome, we want to hear about it and spread the word. Email us at scctmpresident@gmail.com with the wonderful news, and we will see that others know it also.

I hope you enjoy this edition of the Mathmate. Our many thanks are given to Gina Dunn and her committee for publishing it for us. We thank you for sharing your time, talent, and knowledge with the students of our state. Maybe we'll see you at the beach!

Take care,

Jennifer Wilson, NBCT
SCCTM President

Announcements

Upcoming Conference Information and Deadlines:

SCCTM 2014 Annual Fall Conference, Myrtle Beach, South Carolina,
November 6 – 7

[Register by October 18, 2014](#). Registration requests received after October 19, 2014 will be processed at the Later Registration rate.

Membership News:

[Renew your NCTM membership online](#) and designate *South Carolina Council of Teachers of Mathematics* for the affiliate rebate.

<p>If you would like your announcement to appear in the next issue of THE MATHMATE, please email all information to SCMathMate@gmail.com by January 1, 2015. Announcements will be published at the discretion of THE MATHMATE Editorial Board.</p>

Being a Member is NOT Enough!

Marc Drews

On Friday, March 14th, at a minute before 2:00 (3.14159), I walked to my car as a newly-retired state employee. It was a great feeling and, as I reflected on my career that weekend, I realized that nothing helped me grow more as a professional than being a part of the South Carolina Council of Teachers of Mathematics.

As a student at the College of Charleston back in the 70's, I was one of the lucky ones that had Dr. Louise Smith as my professor and mentor. She taught me to see the beauty of mathematics and she convinced me to teach it with love for both the content and the students. As a young teacher, she strongly advised me to become a member of the SCCTM that had been recently created after the merging of two state organizations of math teachers. She also began asking me to do workshops for teachers in the Charleston area.

Seven years into my teaching career, Dr. Smith encouraged me to apply for a position as a mathematics consultant for the Charleston County Schools. When I was named, she said, "What distinguished you from the other candidates was your being a member of the SCCTM and CCCTM (local affiliate) and having experience working with other teachers."

That had a profound impact on me. Louise had been recently elected to the NCTM and was a strong advocate for teachers to take an active role in professional organizations saying that, "being a member is not enough--anyone can join--being involved and being active in the organization is what's most important."

It wasn't long before I was asked to serve as the editor of *The Mathmate*, SCCTM's newsmagazine, a role that I proudly held for eleven years. In 1987, Louise called me again asking if I were interested in a position at the SC Department of Education. She added that she was approached by a friend at the Department wanting leads and she said that "because of my involvement and statewide network of colleagues, I'd be ideal." She helped open another door for me.

Being a member of the organization provided me with tremendous opportunities to present and share nationally, regionally, and throughout South Carolina. When I was named the Director of the Statewide Systemic Initiative in early-1996, I attributed my involvement with SCCTM as the contributing factor because of the connections and relationships established as a member.

As a member of SCCTM, I was able to be involved in several conferences, as a presenter and planner. I even had the chance to coordinate a mini conference whose sessions focused on using children's literature to enhance the teaching of mathematics. It was a career highlight to spend time with the likes of David Schwartz, Louis Sacher, and David Whitin.

As a member of SCCTM, I had the chance to connect Australia's leading math consultant, Charles Lovitt, with our organization. His presentations led to an interest and eventual implementation of Maths Recovery initiative in Oconee County, as well as a major push to utilize math kits, which was later realized through the outstanding work of the AOP Hub.

Two years ago, when I joined the team at EdVenture, one of the first things I did was reconnect with my SCCTM colleagues to help support our work. Together, we hosted a Mathematics Education Night at the museum where teachers gathered to build giant soma cubes and pentominoes, as well as to make polyhedra out of balloons. We also collaborated in a "Twenty Years of STEM Reform" event in September 2013, where leaders who were instrumental in the creation of the regional mathematics and science Hubs gathered, shared stories, and discussed the next steps in STEM reform in South Carolina.

My wife and I are the parents of three adult children, two of whom are teachers. When they entered their profession, I reiterated the message shared with me over a quarter of a century earlier by Dr. Smith, get involved in your professional organization-- it will make all the difference in your career. Both took my advice, our son recently completed a two-year stint as the president of the SC Art Educators Association and our daughter is currently a regional rep for the same organization.

My advice to all teachers is simple: get involved. The possibilities rest in their hands, hearts, and minds. I will forever be humbled by being honored by the organization as one of its recipients of the Outstanding Contributions to Mathematics Education and I can't help but be reminded of the encouragement of Louise Smith.

My hope is that more heed the wisdom of those like Louise. It is the responsibility of all SCCTM members to encourage membership, as well as participation--whether as committee members, training opportunities, presenting at conferences, or serving on the Board. Over the past ten years, I've noticed a tremendous decline in memberships of professional groups all over the state. Lately, membership numbers have been episodic and the drop parallels an apparent decline in the enthusiasm and passion for teaching. That can change with our persuasive leadership.

Everything is cyclic (being the reason I retired on Pi Day). Forty years ago, a mentor encouraged me to be a member... an active member of a professional organization. Now, it's my turn to do the same. I know that SCCTM has had a tremendous impact on my career and am confident it can do the same for all teachers. We need to be proud of our profession and one place where I have always felt proud was with my involvement with SCCTM.

An Implementation of the Tabor Rotation Method In an Elementary Math Methods Course – Artifacts, Student Feedback, and Instructor Impressions

Gary Bradley
University of South Carolina, Upstate

Abstract

The Tabor Rotation Method is a variation of the small group instruction method placing students into four small groups that rotate through four learning stations. This teaching method was introduced to preservice teachers in an Elementary Math Methods Course. The preservice teachers created artifacts, activities, and resources that were common core based for each of the four learning stations. The preservice teachers believed that the Tabor Rotation Method was an effective model of small group instruction although they voiced some concerns. Elementary Level Teacher Educators may want to consider introducing the Tabor Rotation Method as an enhancement to their small group instruction methods.

Introduction

Tabor has presented her Tabor Rotation Method at several SCCTM conferences and workshops. She states that her teaching method guides the cooperative learning strategy, optimizes the student-teacher ratio, and utilizes cooperative and collaborative learning. Additionally, she states that her teaching method naturally differentiates instruction while addressing multiple intelligences and various learning styles (Tabor, 2013). Tabor's workshop session leads participants through a set of steps that creates and implements Common Core based lessons for each of the method's four learning stations. An instructor of a mid-sized university in the southeast who attended her workshop worked with Tabor to implement a pilot study of this instructional model as part of an Elementary Math Methods course.

The instructor introduced the Tabor Rotation Method to an Elementary Math Methods course of 24 preservice teachers. They had a generally positive impression of this instructional method in creating and presenting the artifacts, activities, and resources. However, they did raise questions about problems that they might experience with a full implementation of this instruction method in their clinical settings. Some of these concerns may stem from differences between the Tabor instructional method of small group learning and the direct instruction method often demonstrated by cooperating teachers in preservice teacher's clinical placements.

Overview of the Tabor Rotation

The Tabor Rotation Model is an extension of the small group model of instruction. Four student groups rotate one to three times (depending on the teacher's preference) during each class period. The teacher determines who is in each group and chooses a group leader. The group leader is typically a student who understands math well and is willing to spend a few minutes before or after class talking about the content in each station. The group leader is given an answer key to the assignment in each station. The type and purpose of the assignments vary with each station.

Station 1 – "Teacher Time" gives teachers the time to work directly with their students. The teacher determines what the students' needs are and then uses the most effective materials and instructional method to meet those needs. Instructional materials can include textbooks, literature books, multi-media, manipulatives, and paper/pencil activities. The instructional method can include direct instruction, instructional media, or working in pairs. Station 2 – "Applications" gives students the opportunity to apply the skill set they have been learning in the other three stations. Applications include math journals, interactive online resources, textbook, workbook, and study sheet based assignments. Station 3 – "Manipulatives" uses concrete objects such as unifix cubes, base ten blocks, dice (number generators), attribute blocks, and plastic coins. These objects are used to create concrete examples of the abstract terms, symbols, and operations. Station 4 – "Games" station encourages students to sharpen their math skills, and review math concepts. Some games have students compete with the clock or themselves. The materials needed for the station can consist of teacher-made games or everyday math game kits. Math-focused games on the internet or iPod applications are often used at this station.

Research on the Tabor Rotation Method

Cooperative learning, a foundational component of the Tabor Rotation Method, is strongly supported by the research. Cooperative learning gives teachers an easier, more productive, and more enjoyable approach to teaching by combining theories of social psychology and classroom practice (Johnson & Johnson, 1987). These theories are based on the Piagetian and Vygotskian social learning theories involving individual knowledge acquisition (Palincsar, 1998). Cooperative learning is based on the idea that an individual can succeed only when the whole group succeeds (Johnson & Johnson, 1999). This idea helps students see each other more for their potential to help the group and less for their background, race, or social status.

The Tabor Rotation method supports diversity in the classroom. Slavin (1980) and Baer (2003) found that cooperative learning methods increase student achievement, improve race relations in desegregated schools, positively impact mutual concern among students, and improve student self-esteem. Students with diverse learning styles and ability levels are rewarded by the Tabor Rotation method (Holloway, 2011). The four learning stations of the Tabor method address the learning styles of the visual, auditory, and the kinesthetic learner.

The small group model of this teaching method can lower the individual student's anxiety in a math classroom. Many students don't ask questions in a regular classroom because they don't want to look foolish to their peers. However, these same students will be more likely to ask a question in a small group (Faust & Paulson, 1998). Small groups facilitate a cooperative learning atmosphere where students feel freer to ask and answer questions.

The Tabor model further encourages active student learning by assigning hands on math related tasks. These hands on tasks encourage active learning using materials and lessons designed to help students understand mathematical concepts (Moch, 2002). Hands on materials, such as unifix cubes, base ten blocks, dice (number generators), attribute blocks, or plastic coins are often used. Preservice teachers used the large array of manipulatives to create concrete models of abstract math symbols and operations.

Implementation of the Tabor Rotation in an Elementary Math Methods Course

The Tabor Rotation method was introduced into the curriculum of the Elementary Math Methods Course. The Tabor method uses small groups with individual responsibilities (Johnson & Johnson, 1999) but takes the role of the group leader to the level of group mentor. The group leader or mentor facilitates each group members' assigned responsibility and focuses work around the learning task. The student who takes this responsibility will need to spend 5 to 15 minutes outside of class with the teacher going over the problems that they will work on the next day. The group leader is also given a copy of the answer key. Students who understand math well, have good interpersonal skills, and who are willing to meet with the teacher outside of class have the qualities of a good group leader. Tabor suggests that these group leaders only be changed a few times each year (Tabor, 2013).

The preservice teachers were then introduced to the functions and goals of the four stations. They were also given several examples of effective lessons for each station. Standards based activities, artifacts, and resources in the form of short 10 minute lessons were created by the preservice teachers. They presented these lessons to their peers who played the part of elementary students. Group feedback on the material and presentation was encouraged and facilitated refinements in the lessons. Lessons that were rated by the preservice teachers as both effective and enjoyable are listed in the sections below.

Activities and Artifacts Produced by Pre-Service Math Teachers

Station 1 – “Teacher Time” Teachers work directly with the student during teacher time. Teachers can use this time to review skills and terms, introduce a new lesson, or whatever the teacher thinks would best help their students. The preservice teachers were asked to introduce a new math skill or concept using a math story book of their choice from the library. This facilitated the valuable component of literacy into the math classroom (Kowalski, Prett-Frontczak, & Johnson, 2001). Each preservice teacher was asked to bring in two elementary level books and determine what Common Core State Standards the book addressed. They presented the book and shared how they might use the book to engage their students. The following are some of the books that the preservice teachers found especially winning.

- *Alexander, Who Used to be Rich Last Sunday* (Viorst, 2012) is a story about a boy who learns how quickly money can be spent. Preservice teachers used play money to follow along with the book in finding out how much money Alexander had after each one of his purchases. Preservice teachers stated that this book would be helpful for teaching or reviewing subtraction or money skills.
- *The Wing on a Flea* (Emberley, 2001) sets shapes in bright colors against a black background. Preservice teachers saw this book having the potential to inspire elementary students to see geometric shapes in their environment and identify the names of the shapes. Preservice teachers demonstrated how students could cut geometrical shapes from colored construction paper to form artistic shape collages.
- *Eat Your Homework* (McCallum, 2011) introduced more challenging math concepts in a humorous way. Preservice teachers were drawn to the author’s simple illustrations of pi, tessellations, fractions, and Fibonacci numbers. They also noted the numerous hands on activities on these topics throughout the book.
- *Roman Numerals* (Adler, 1977) was recognized by preservice teachers to be an engaging introduction to Roman Numerals. The book explores the development of the Roman numeral system and its relevance in today’s society. The book includes hands-on exercises in logic, mathematics, and numerical systems.
- *How High Can a Dinosaur Count?: and Other Math Mysteries* (Fisher, 2006) wasn’t about dinosaurs, but preservice teachers enjoyed the book’s 15 “math mysteries.” Each number puzzle combined a brief vignette and question with illustrations that provided additional information and visual hints.
- *Math for All Seasons* (Tang, 2002) encouraged children to think through problems rather than to rely on formulas and memorization. Preservice teachers said that the book could also help students in transition from counting to computation by introducing ways to group and add numbers.
- *Grapes of Math* (Tang, 2001) explores mathematical word problems which require students to invent their own problem solving strategies. Preservice teachers liked how this book helped students think about grouping strategies used for mental math in a visually rich and appealing way.

Several preservice teachers recognized the long term educational value of books. One preservice teacher stated, “There is nothing like a good book to get students’ attention.” Many of the students said that they had a difficult time choosing just two books for this station because there were so many wonderful books that they found. Station 1 of the Tabor Rotation Method helps teachers to incorporate literacy into their math lesson. It also is an effective way to introduce a math concept to students in a small group setting.

Station 2 – The “Application” station is where students apply the concepts that they have learned while in the other stations. Applications include math textbooks, workbooks, study sheets, journals, and online resources. Preservice teachers were asked to find or develop activities that met elementary Common Core Math Standards. They found numerous activities and presented them to the other preservice teachers in the class. The applications listed below are ones that they especially liked.

- Preservice teachers employed activities that developed number sense. A string is tied between two points such as the two ends of a dry erase board. The preservice teacher defined the beginning and ending point of the string (i.e. 0 to 1 for fraction cards and percentages). The students were given a percent, fraction, or decimal written on a number card. Students were then asked to attach their card with a paper clip to the point on the string that best approximated the card’s position on the number line. Students in the classroom then discussed if they agreed or disagreed with the location where the card was placed.
- Preservice teachers helped their colleagues who served as students to create a classroom treasure map. The center of the room had an “X” taped on it. North and South (marked in tape above and below respectively to the “X”) was the positive and negative y-axis. East and West (marked in tape right and left respectively beside the “X”) was the positive and negative x-axis. Each student was asked to select an object in the room to be the treasure. Students then created the treasure map as a series of directions and number of steps starting from the center “X.” Once the map was completed, a student from a different group was asked to “find the treasure.” If the treasure was successfully “discovered” both the treasure finder and the map maker were positively rewarded. The preservice teachers recommended that the total number of directions should correspond to the grade level (i.e. 5th grade had a total of 5 directions).
- Preservice teachers asked their students to use a standard instrument (meter stick) and then a non-standard instrument (the length of a new pencil) to measure lengths of items in the classroom. Doors, windows, and desks were the first to be measured. The activity became more challenging when preservice teachers asked their students to measure the lengths of things that could not be directly measured, such as the height of the wall or the area of the ceiling. Most students quickly figured out that measuring the length and width of the room and then calculating the area of the floor will give the same area as the area of the ceiling. Students also found that measuring the height of one cement block and then multiplying by the number of blocks in the wall would give the wall’s height. Preservice teachers enjoyed watching their students work through this problem solving process.
- Sports statistics abound in the newspaper and the internet. Preservice teachers developed activities that guided their students in using these statistics to calculate percentages, mean, median, and mode. One preservice teacher asked the question, “If the Braves win two of their three games they are scheduled to play this weekend, what will be their overall percentage of wins to losses?” Preservice teachers also developed follow up questions that required additional logic and calculation such as, “How many games would the number 2 ranked Braves need to win and the number 1 ranked Phillies need to lose in order for the Braves to be back in first place?” Students used statistics listed on-line or in newspapers to find the team’s current data and then create a statistical calculation table for the two teams. Although this activity involved multiple steps, preservice teachers believed their students would enjoy this activity because it could be used with any of their favorite baseball teams. Data from other sports such as basketball, hockey, and NASCAR were used to create similar tables, graphs, and charts. Students enjoyed putting pictures of players, drivers, and team logos on their charts or graphs.
- The preservice teachers made math flash cards from 3x5 note cards. The flash cards show the conversion of numbers from fractions to decimals or percent values. Analog time and digital time flash cards also worked well. The time or number would be written on one side of the card and the conversion would be written on the other side. The preservice teachers demonstrated how they would use these cards as reinforcement, review, or as formative assessment.
- The preservice teachers used math journaling as an activity that would reinforce the lesson. The journaling process of putting math concepts into words requires students to move from the left brain function of math calculations into the right brain function of math reasoning and comprehension (Burns, 1995). The preservice teachers created guiding questions for their students and presented them at the end of their lesson. The preservice teachers recommended that the sentence response should correspond to the grade level (i.e. 3 sentences for grade 3).

Station 3 – The goal of the “Manipulatives” station is to give students the opportunity to create concrete examples of the abstract terms, symbols, and operations. Objects such as unifix cubes, base ten blocks, dice (number generators), attribute blocks, or manipulatives created by the preservice teachers. These manipulatives can be made at little or no cost other than teacher preparation time. Activities in station 3 often use artifacts that students can make and then take home. Parents/caregivers can see what their children are learning in math class as well as have the opportunity to use the manipulative for additional math concept reinforcement. Preservice teachers created Common Core based activities for this hands on focused station. They then presented these hands-on activities to their colleagues who played the part of the student. The following activities received the most positive feedback from the class.

- Preservice teachers put 20-50 old buttons in a box. Then they asked their students to sort the buttons using attributes such as size, color, or buttonhole size. Students wrote down the attributes that they used to sort the buttons and the number of buttons in each group. The preservice teachers then asked their students if there might be other attributes that they could have used to sort the buttons. Preservice teachers used this activity with a variety of other classroom objects such as pencils, erasers or even the color of shoes the students wore.
- Preservice teachers engaged their students in a game of concentration with a set of cards that they had made. A set of 24 matched cards were created. Preservice teachers wrote a fraction on one card and a matching decimal on a second card. The backs of both cards were left blank. The cards were then shuffled and arranged upside down in a 4 by 6 matrix. Students selected a card and then tried to find its match. If the match couldn't be found then both cards were replaced and turned upside down. The partner selected other cards while trying to remembering the cards shown during the partner's turn. Players kept their matches and then tallied their cards at the end of the game. Additional sets of cards were created with matching coins and their dollar value, decimals and percentages, as well as shapes with their names.
- Preservice teachers wrote a set of letter patterns on a Promethean Board. They asked their students to match the letter patterns with block patterns using attribute blocks (plastic colored blocks). Preservice teachers checked to see if their students had accurately reproduced the letter pattern with the colored block pattern. The level of thinking increased when the preservice teachers used abstract symbols in the pattern set. Students were then called to the board to make up their own letter or symbol patterns and to have the class try to match the pattern.
- Preservice teachers gave their students objects such as balls, ice cream cones, kites, cans, pyramids, and boxes and asked them to write the name of the objects. Students were then asked to count the number of sides, edges, and corners of these objects and write this information beside the objects' common name. Finally, students were asked to identify the objects' geometric name (sphere, cube, cylinder, prism, and rectangular prism). Students were then asked to find other objects in the classroom that matched the objects' attributes.
- Preservice teachers gave their students 3 flat sticks (i.e. Popsicle sticks) and two crayons (i.e. red and blue). Two of the sticks were colored red on one side and the other side was left blank. The third stick was colored blue on one side and left blank on the other side. Students were asked to hold all three sticks in one hand above the desk and drop the sticks. Each drop was given a score based on the orientation of the three sticks. When all plain sides land face up students get 4 points. When all colored sides land face up students get 4 points. When two red and one plain land face up students get 6 points. When two plain and one colored land face up students get 6 points. When one plain, one red, and one blue land face up students get 2 points. Preservice teachers had their students add up their scores after each drop. Students tried to get exactly 50 points in the fewest number of drops.
- Preservice teachers used web pages such as <http://www.superteacherworksheets.com/solid-shapes.html> to download 3D nets of shapes and print them out on multi-colored construction paper. Students cut out the shapes and glued the edges together. (<http://www.korthalsaltes.com/cuadros.php?type=p>) was used to create more advanced shapes. Students enjoyed creating, identifying, and playing with their 3D shapes.
- Preservice teachers shared a wealth of virtual manipulatives and activities using The National Library of Virtual Manipulatives (<http://nlvm.usu.edu/en/nav/vlibrary.html>). This site has dozens of online manipulatives and activities for K-12 in the areas of Numbers, Operations, Data Analysis, Probability, Measurement, and Geometry. The activities and manipulatives include the corresponding Common Core Standards.

Station 4 – The “Games” station helps students sharpen their math skills, and review math concepts. Since an increasing number of school systems are using iPad and iPod technology in their classrooms (Murray & Olcese, 2011), preservice teachers need to have experience evaluating and incorporating the math applications these devices use. Each preservice teacher used an iPad to evaluate 5 different math applications. Common Core alignment was included in the evaluation. All of the iPad applications were selected from a search of the iTunes App Store using the key words “free” and “elementary math.” This list was further refined by looking at the user rating, user comments, and number of downloads. Over a hundred iPad applications were considered, most of which were free. The following apps were given the most favorable reviews by the preservice teachers.

- *3 Dimensional Math Racing* by POTG Apps. This application is an off-road truck racing game. The faster the student answers the math questions the faster the truck goes. Graphics, music, and sound effects make this game a favorite. Teachers or students can choose the level of difficulty and the operations (add, subtract, multiply, and divide).
- *4th Grade Math: Splash Math Worksheets (HD Lite)* by Study Pad. Splash Math is a collection of fun and interactive math problems aligned to Common Core Standards. Preservice teachers noted the characters, graphics and music created a positive atmosphere for math review and reinforcement. Student progress can be tracked for speed and accuracy.
- *Abby Monkey: Spring Math* by 22learn. Students select an animal avatar and then play a series of math games that are designed to review the four basic operations. The graphics, characters, and music give this game a cheerful feel. Difficulty levels can be selected. Accuracy and speed is charted by user login name.
- *Mad Math Lite* by Reese McLean. Mad Math is a basic flash card game, but what sets it apart is the control of this application. Users can select the biggest number in the flash cards (from 4 to triple digits) the operation (four basic operations), the number type (whole, decimal, mixed), positive and negative numbers, and the number of flash cards. Student progress is charted in the statistics box.
- *TanZen HD Lite* from Little White Bear Studios. This application is a Tangram puzzle game that is easy to play. Students can choose from 48 puzzles and then fit all 7 game pieces within the shaded area without overlapping. Students translate, rotate, and flip the game pieces to fill in the shape outline. Students can ask for hints when they get stuck. TanZen will color and animate the puzzle when it is completed.
- *Sushi Monster* by Scholastic. Feed the Sushi Monster from a set of dishes with numbers on them that are the sum of the number hanging on its neck. The graphics and Japanese music make this game enjoyable. Students can also choose to play this game using products instead of sums. Difficulty increases with each additional monster. This game increases student’s use of mental calculation of sum and fractions.
- *Brain Pop Math* by Brain Pop. This application introduces students to concepts such as ratio, proportion, percent, geometry, measurement, and probability. The main character is a loveable robot who is often doing something funny. Each section has a text-based explanation of the topic along with examples. Preservice teachers suggested that this app would be a good way to introduce a new math concept to students.
- *Amazing Coin (USD) Free* by Joy Preschool. Amazing Coin is a set of games that uses USD coins to count, pay, and make change. A quarter rewards each correct answer. Students can use their quarters to buy food items in the store. The graphics and positive verbal reinforcement make this an enjoyable game for students to review their mental addition and subtraction skills while playing with money.

The preservice teachers enjoyed reviewing the iPad applications for Station 4, “Games.” They discovered no one application had the perfect combination of graphics, music, characters, and activities. One preservice teacher wrote, “The music on this game drove me crazy. If I were a student I wouldn’t want to play this game for more than just a few minutes.” Students noted the wide variation in the quality of the applications. One student summarized their review of their application by saying, “This application is boring and not at all useful. Don’t download this free application.” Another student wrote about a different application, “This application is amazing. Get it while it’s still free.”

Preservice Teacher and Instructor Impression of the Tabor Rotation Method

The lessons, artifacts, activities, and resources listed above were selected, evaluated, and presented by the preservice teachers. Preservice teachers presented these lessons to their colleagues in several rounds of the Tabor Rotation Method. The class participated in 9 rounds of the Tabor Rotation during the entire semester. The preservice teachers started referring to the station lessons as “mini-lessons.” At the end of the course, the preservice teachers filled out an exit survey. The student responses were overwhelmingly positive. One student said, “the ‘mini-lessons’ were the best part of the class.” Another student wrote, “I will be using activities from the ‘mini-lessons’ in my clinical experience.”

Although preservice teachers were very positive about the Tabor Rotation Method they did voice some concerns. Several students said they didn’t like the extra responsibilities given to the group leader. Students questioned the necessity of training time for the group leader during, before, or after school. One student said, “My cooperating teacher and I leave right after school. I don’t know when we could meet the student group leaders outside of class.” Many of the preservice teachers felt that they could keep each group on task without the need of a group leader. However, Tabor states that having a group leader gives group members a peer to relate to and gives the teacher the opportunity to help other students.

Several preservice teachers were surprised when they were told that the group leader would be given the answer key to the activities in all four stations. They thought that the group leader would just give the members of their group the answers. Tabor shared in her workshop (Tabor, 2012) that teachers often have this question. She used the illustration of how the fastest runners in Physical Education class are careful to report the correct time elapsed for their running events. The slower runners respond with a more ambiguous answer about their times. Tabor believes that students who do well in math will make good group leaders. They also will want to see the members in their group succeed.

Other preservice teachers struggled with the format of the Tabor Rotation Model. A preservice teacher spoke up at the end of a Tabor lesson and said, “I really don’t get the whole idea of the Tabor Rotation. How am I supposed to teach my class with four separate things happening at the same time?” This student may have voiced their concern because the math classes they have attended or observed in their clinical settings so often use the direct instruction method. Tabor (2012) shared that her model gives students the opportunity to learn from the group leaders and each other. This frees the teacher to focus on the specific needs of small groups of students. Asking preservice teachers to transition from a direct instruction mindset to the Tabor Rotation mindset is not always an easy change for them to make.

The Tabor Rotation Method did facilitate active learning of preservice teachers by playing math games and using math manipulatives. It enhanced differentiated learning and gave each student the opportunity to succeed within the group. This teaching method incorporated literacy, problem solving skills, concepts reinforcement, and creativity. Preservice teachers stated that they enjoyed developing and presenting the lessons and looked forward to sharing these lessons with their students in their clinicals. However, instructors of preservice elementary teachers may need to spend additional time modeling cooperative learning strategies for their preservice teachers who may have little exposure to this teaching method. Math teacher educators may want to consider using the Tabor Rotation Method in their Elementary Math Methods course as an effective teaching model.

References

- Adler, A. D., & Barton, B. (1977). Roman numerals. New York, NY: Crowell.
- Baer, J. (2003). Grouping and achievement in cooperative learning. *College Teaching*, 51 (4) 169-174.
- Burns, M. (1995). Writing in math class? Absolutely. *Instructor*, 104(7), 40-47.
- Emberley, E. (2001). The wing on a flea. New York, NY: Little, Brown Books for Young Readers.
- Faust, J. L., & Paulson, D. R. (1998). Active learning in the college classroom. *Journal on Excellence in College Teaching*, 9(2), 3-24.
- Fisher, V. (2006). How high can a dinosaur count?: And other math mysteries. Schwartz & Wade.
- Holloway, J. H. (2001). Grouping students for increased achievements. *Educational Leadership*, 59(3), 84-85.
- Johnson, D. W., & Johnson, R. T. (1987). Learning together and alone: Cooperative, competitive, and individualistic learning. Upper Saddle River, NJ: Prentice-Hall, Inc.
- Johnson, D. W., & Johnson, R. T. (1999). Making cooperative learning work. *Theory into practice*, 38(2), 67-73.
- Kowalski, K., Pretti-Frontczak, K., & Johnson, L. (2001). Preschool teachers' beliefs concerning the importance of various developmental skills and abilities. *Journal of Research in Childhood Education*, 16(1), 5-14.
- McCallum, A. (2011). Eat your math homework: Recipes for hungry minds. Watertown, MA: Charles Bridge Publishing, Inc.
- Moch, P. L. (2002). Manipulatives work!. *The Educational Forum*, 66(1), 81-87.
- Murray, O. T., & Olcese, N. R. (2011). Teaching and learning with iPads, ready or not?. *TechTrends*, 55(6), 42-48.
- Palincsar, A. S. (1998). Social constructivist perspectives on teaching and learning. *Annual review of psychology*, 49(1), 345-375.
- Slavin, R. E. (1980). Cooperative learning. *Review of Educational Research*, 50(2), 315-342.
- Tabor, G (2012, October). Tabor rotation: the proven framework for simplifying small-group, differentiated instruction. 2012 South Carolina Council of Teachers of Mathematics Fall Conference, Myrtle Beach, South Carolina.
- Tabor, G. (2013, March 18). Glenna Tabor News. Retrieved March 20, 2013 from <http://glennatabor.com/new/>.
- Tang, G. (2001). *The grapes of math*. New York, NY: Scholastic Press.
- Tang, G. (2002). *Math for all seasons*. New York, NY: Scholastic Press.
- Viorst, J. (2012). *Alexander, who used to be rich last sunday*. New York, NY: Atheneum Books for Young Readers.

Author:

Gary Bradley is an Assistant Professor of Education at the University of South Carolina, Upstate. He is a former middle school mathematics teacher and has interests in instructional technology and math education.

Word Numbers Fortified by Universal Design for Learning

Diana Cheng
Towson University

Nicole Horner
George Washington Carver Center for Arts and Technology

Abstract

In this article, we show how a letter-number substitution problem, “FOUR + ONE = FIVE” can be used in the elementary and middle school classroom. We describe ways of scaffolding activities related to this problem using the Universal Design for Learning principles of providing multiple modes of representation, explanation and expression. We also provide suggestions for assessment using extensions to this problem.

The Center for Applied Special Technology’s (CAST) Universal Design for Learning (UDL) Guidelines Version 1.0 (2008) suggest that teachers design their instruction in a way that anticipates and meets the diverse needs of students. Specifically, the guidelines suggest that teachers provide multiple modes of representing knowledge so that learners will have their choice of different ways of learning information, multiple ways of expression so that learners have a range of ways they can demonstrate what they know, and engage learners such that what they are learning interests them and challenges them appropriately. In this paper, we illustrate how the UDL principles were used to create an upper elementary/middle school mathematics lesson with the learning goal that students will apply their knowledge of addition, place value, and substitution to evaluate expressions and find solutions to a letter-number substitution equation.

According to the Common Core State Standards (CCSSI, 2011) mathematics content standards for elementary and middle school curriculum, students should understand the difference between face value and place values of digits in multi-digit numbers (2.NBT.1, 5.NBT.1), use their place value understanding and properties of operations to perform multi-digit arithmetic (3.NBT.2, 4.NBT.4), and evaluate expressions in which letters stand for numbers (6.EE.2). Students should also be able to determine whether a given number makes an equation true (6.EE.5), and understand that a variable can represent an unknown number or any number in a specified set (6.EE.6). We decided to use this letter-number substitution problem as an engaging way for students to enact this content. The letter-number substitution problem we chose is “FOUR + ONE = FIVE” as written below:

$$\begin{array}{r} \text{FOUR} \\ + \text{ONE} \\ \hline \text{FIVE} \end{array}$$

Figure 1. “FOUR + ONE = FIVE” Letter-number substitution problem.

Letter-number substitution problems involve students' understanding and applying the following rules for an equation (Fendel et al, 1999):

- Each distinct letter represents a different single-digit number from 0 to 9. Note that the letter F appears twice, and each time it appears it represents the same number. The letters F, O, U, R, N, E, I, and V all represent different digits.
- A letter representing zero never starts a multi-digit number. This means that F and O will never be equal to zero.
- Each letter represents the face value of the number and is written in the number's place value – for instance, FOUR represents the number computed by $F*1000 + O*100 + U*10 + R*1$.

Representation

In order to provide students with multiple means of representing the problem, we created two tools as alternatives to pencil-and-paper: clear plastic sleeves and a Microsoft Excel spreadsheet.

Clear plastic sleeves representation

The first tool we created is a physical manipulative with a sticker of each different letter in the equation mounted on a differently colored square piece of paper. This visually highlights the variables which are the same (for instance, the two papers on which the letter F appears have the same color) and the variables which are different (for instance, the paper on which the letter F appears is different from the paper on which the letter R appears). The different colors of paper have different shading or designs in case color-blind students are using this tool. The square pieces of paper were then inserted into a coin collector's clear plastic sleeve to represent the fixed variables. For students who are more significantly visually impaired, teachers can tape the colored papers over the front of the plastic sleeves so that the students can feel the shape of the raised stickers of each letter. See Figure 2 below for a picture of the clear plastic sleeves.



Figure 2. Clear plastic sleeves and colored paper to represent $FOUR + ONE = FIVE$.

To try out different variables to substitute as the letters, students had the option of using dry-erase markers on the clear plastic sleeves or digits mounted onto square transparencies (similarly friendly for use with the visually impaired, as the stickers for the digits are raised above the transparency surfaces). Both of these options allowed students to use trial and error to easily move around and replace digits representing the letters.

Spreadsheet representation

The spreadsheet we created has three templates, with each subsequent template having increased scaffolding than the prior template. On the first template, students are simply provided the letter-number substitution problem and the given set of digits 0-9 on the right hand side. Students using this have the option of copying and pasting digits into the cells of the spreadsheet where they intend to substitute the digits for letters, or directly typing in these digits. Below is a picture of the screen provided to the students on this first spreadsheet template.

F	O	U	R
	O	N	E
+			
F	I	V	E

0	0
1	1
2	2
3	3
4	4
5	5
6	6
7	7
8	8
9	9

Figure 3. Spreadsheet template #1.

The second template of the spreadsheet has a built-in calculator so that when students type in digits representing each of the letters on the left hand side, the numbers representing $F*1000 + O*100 + U*10 + R*1$ and $O*100 + N*10 + E*1$ are added. Students then only need to confirm whether the resulting sum calculated by the spreadsheet is the same as the number represented by the digits which they chose for FIVE.

F	O	U	R
+	O	N	E
F	I	V	E

F	O	U	R
F*1000 + O*100 + U*10 + R*1			
0			
	O	N	E
O * 100 + N * 10 + E * 1			
+			
F	I	V	E
SUM of FOUR + ONE			
0			
F*1000 + I*100 + V*10 + E*1			

Do all of the different letters represent a different digit?
Do the same letters represent the same digit?

Do the two boxes in pink match?
If yes, then it's a solution
If no, then it's not a solution

Figure 4. Spreadsheet template #2.

This second spreadsheet template was designed to for multiple purposes. It provides additional assistance for students who have trouble understanding how the digits' face values connect with their place values in the multi-digit numbers of FOUR, ONE, and FIVE (this refers to Common Core State Standard 5.NBT.A.1). It also helps students whose weaker arithmetic skills may hinder the accuracy of their trial and error process – for instance, students who incorrectly add numbers may think that an incorrect solution that they have is valid or that a correct solution is invalid. The spreadsheet can also be used for more advanced students to double check their work quickly after they have identified solutions.

The third template provides even more scaffolding to help students. Like in the first and second templates, students are to type in digits representing the letters on the left hand side. However, instead of asking students to ensure that each letter represents a different digit, and that the same letters are represented by the same digit, the spreadsheet has the following built-in logic statements whose results are made visible to the students:

- a. Whether F_1 is the same as F_2
- b. Whether O_1 is the same as O_2
- c. Whether F_1 is distinct from N, E, I, V
- d. Whether O_1 is distinct from N, E, I, V
- e. Whether U is distinct from N, E, I, V
- f. Whether R is distinct from N, E, I, V
- g. Whether N is distinct from I, V
- h. Whether E is distinct from I, V
- i. Whether I is distinct from V

If and only if all of the above are true (see left hand columns of Figure 5), and if the sums computed on the right hand side match, then the solution is valid for “FOUR + ONE = FIVE.”

Check: F_1 = F_2?	TRUE
Check: O_2 = O_2?	TRUE
Check: E_1 = E_2?	TRUE
Check: F /= O?	TRUE
Check: F /= U?	TRUE
Check: F /= R?	TRUE
Check: F /= N?	TRUE
Check: F /= E?	TRUE
Check: F /= I?	TRUE
Check: F /= V?	TRUE
Check: O /= U?	TRUE
Check: O /= R?	TRUE
Check: O /= N?	TRUE
Check: O /= E?	TRUE
Check: O /= I?	TRUE
Check: O /= V?	TRUE
Check: U /= R?	TRUE
Check: U /= N?	TRUE
Check: U /= E?	TRUE
Check: U /= I?	TRUE
Check: U /= V?	TRUE
Check: R /= N?	TRUE
Check: R /= E?	TRUE
Check: R /= I?	TRUE
Check: R /= V?	TRUE
Check: N /= E?	TRUE
Check: N /= I?	TRUE
Check: N /= V?	TRUE
Check: E /= I?	TRUE
Check: E /= V?	TRUE
Check: I /= V?	TRUE

F_1	O_1	U	R
5	1	3	0
+	O_2	N	E
	1	6	7
F_2	I	V	E
5	2	9	7

F	O	U	R		
F*1000 + O*100 + U*10 + R*1					
5130					
	O	N	E		
O * 100 + N * 10 + E * 1					
167					
+	F	I	V	E	SUM of FOUR + ONE
5297					5297
F*1000 + I*100 + V*10 + E*1					

Do the letters fit the constraints?	
YES	

Do the two boxes in pink match?	
YES	

Figure 5. Spreadsheet template #3.

This third template might be used by the teacher or students to quickly verify whether a solution is valid or invalid.

Expression

Depending on the tools which students choose to use, students have different options for expressing their findings. Students have the option of using a pencil-and-paper worksheet on which to collect their trials; this worksheet includes a place where students could circle valid or invalid to indicate whether the solution works. (see Appendix A for the worksheet)

Students who use the clear plastic sleeves can take photographs of their responses if they find that transcribing their results is time-consuming or mechanically challenging. Students who use Excel spreadsheets can copy and paste their responses onto separate spreadsheets (for instance, they can create one sheet of valid responses and another sheet of invalid responses).

Engagement

Hunt and Andreasen (2011) suggest that to help students become more motivated in pursuing mathematical tasks, the tasks posed should be connected to real life and situated in interesting contexts. The task of finding solutions to “FOUR + ONE = FIVE” is interesting because it is a mathematical pun: $4+1=5$, and it is possible to find digits representing the letters such that $\text{FOUR} + \text{ONE} = \text{FIVE}$. A real-world application of such letter-substitution problems is cryptography whereby messages received are encoded. In order to decode the messages, substitutions need to be made to reveal the original messages. Students who would like to learn more about cryptography can be encouraged to read books on the history of code-making and code-breaking and thus extend this mathematical activity to an interdisciplinary pursuit.

Assessment

Assessment is a critical component of the UDL guidelines and is used so that teachers can measure to what extent students have achieved the intended learning goals. Below, we provide some questions that could be used for assessment and some sample responses that students could give.

Question 1: Did you notice any shortcuts to finding solutions, and if so, why are these valid shortcuts?

Some possible shortcuts which students might find include the following:

- 1) The letter R must be the digit 0, because in the ones place, $R + E = E$. The reason for this is the zero property of addition.
- 2) We know that $O + O$ cannot produce a carry over to the thousands place, so O must be represented by either 1, 2, 3 or 4.
- 3) The digits representing the letters U and N are interchangeable because of the commutative property of addition. Namely, if you find one set of digits for (F, E, O, I, U, N, V) that produce a valid solution, you can easily find another set of digits for that produce a solution by interchanging the values for N and V.
- 4) The digits representing the letters F and E are interchangeable because neither F nor E depends on the result of another addition problem in another column. Namely, if you find one set of digits for (F, E, O, I, U, N, V) that produce a valid solution, you can easily find another set of digits for that produce a solution by interchanging the values for F and E.
- 5) Students could list out the possible sets of three digits that exist for (U, N, V) for which $U+N=V$ (with no carry to the hundreds place) and $U+N \rightarrow V$ (with a carry to the hundreds place).

Question 2: Given a solution to the $\text{FOUR} + \text{ONE} = \text{FIVE}$ problem, $5130 + 167 = 5297$, find as many different solutions as you can using the same digits. How will you know when you have found all of the possible solutions using these digits?

To rephrase the question, we are given the set of digits, (0,1,2,3,5,6,7,9) and are asked to find all of the possible solutions to “FOUR+ONE=FIVE” using these digits. We can try to find solutions for which $O = 1, 2$, and 3 . There are no solutions for which $O = 4$ because the digit 4 is not available in our specified set.

$5160 + 137 = 5297$, $7130 + 165 = 7295$, and $7160 + 135 = 7295$ are valid solutions by changing the orderings of the digits representing letters F, E, U, N. There are no other solutions in which $O = 1$ and $U+N \rightarrow V$ with no carry to the hundreds place.

The solutions for which $O = 1$ and $U+N \rightarrow V$ with a carry to the hundreds place are the following:

- $6150+179=6329$, $9150+176=9326$, $6170+159=6329$, $9170+156=9326$
- $2160+197=2357$, $7160+192=7352$, $2190+167=2357$, $7190+162=7352$
- $2170+195=2365$, $5170+192=5362$, $2190+175=2365$, $5190+172=5362$

There are no solutions for which $O = 2$ and $U+N=V$ without a carry to the hundreds place, because the digit 4 is not available in our specified set (for $O+O = I$, we would need $2+2=4$).

The solutions for which $O = 2$ and $U+N \rightarrow V$ with a carry to the hundreds place are the following:

- $1260+279=1539$, $9260+271=9531$, $1270+269=1539$, $9270+261=9531$

The solutions for which $O = 3$ and $U+N=V$ without a carry to the hundreds place are the following:

- $1320+359=1679$, $9320+351=9671$, $1350+329=1679$, $9350+321=9671$

The solutions for which $O = 3$ and $U+N \rightarrow V$ with a carry to the hundreds place are the following:

- $2350+369=2719$, $9350+362=9712$, $2360+359=2719$, $9360+352=9712$
- $5320+396=5716$, $6320+395=6715$, $5390+326=5716$, $6390+325=6715$
- $1360+392=1752$, $2360+391=2751$, $1390+362=1752$, $2390+361=2751$

Question 3: Create your own letter-number substitution problem and find its solution(s).

Some letter-number substitution addition problems which involve fun words created by the authors' students include:

HERE + SHE = COMES ($9,454 + 894 = 10,348$); DAYS + TOO = SHORT ($9,871+655=10,526$);
SATURN + URANUS = PLANETS ($546,790+794,075= 1,340,865$); HEAD + HAND = KNEE
($4,513 + 4,653 = 9,166$); ORANGE + GREEN + RED = YELLOW ($648,950 + 54,009 + 402 = 703,361$).

Discussion

We chose to design a UDL-based lesson on the “FOUR+ONE=FIVE” letter-number substitution problem because it has many solutions – in fact, it has 1200 solutions (Authors, under review). We allow for students' creativity by not prescribing a particular algorithm by which students are to find solutions. We ask students to record their valid and invalid solutions so as to give students a chance to demonstrate their progress on finding solutions. Students who easily discern patterns and properties of addition may find shortcuts to generate solutions to the problem, whereas students who do not immediately recognize these patterns will still be able to make progress on finding solutions. Students have several options for representing the problem – including using clear plastic sleeves and a spreadsheet. They have several options for demonstrating their solutions – including using a worksheet, using photographs, and using a spreadsheet. They have several options for checking their solutions – including using a calculator or using a more scaffolded spreadsheet with built-in logical statements that check the validity of their solutions. Options for assessment questions are provided with sample responses.

Appendix A. FOUR + ONE = FIVE worksheet.

F	O	U	R
+	O	N	E
F	I	V	E

Valid/Invalid

F	O	U	R
+	O	N	E
F	I	V	E

Valid/Invalid

F	O	U	R
+	O	N	E
F	I	V	E

Valid/Invalid

F	O	U	R
+	O	N	E
F	I	V	E

Valid/Invalid

F	O	U	R
+	O	N	E
F	I	V	E

Valid/Invalid

F	O	U	R
+	O	N	E
F	I	V	E

Valid/Invalid

F	O	U	R
+	O	N	E
F	I	V	E

Valid/Invalid

F	O	U	R
+	O	N	E
F	I	V	E

Valid/Invalid

References

- Authors. (under review). Finding the total number of solutions to “FOUR + ONE = FIVE”: Word sum problems for elementary through graduate students.
- Center for Applied Special Technology. (2008). National Center on Universal Design for Learning: UDL guidelines 1.0. <http://www.udlcenter.org/aboutudlguidelines>.
- Common Core State Standards Initiative (CCSSI). 2010. Common Core State Standards for Mathematics. Washington, DC: National Governors Association Center for Best Practices and the Council of Chief State School Officers. http://www.corestandards.org/wp-content/uploads/Math_Standards.pdf
- Fendel, D., Resek, D., Alper, L., Fraser, S. (1999). Interactive Mathematics Program: Year 3. Key Curriculum Press: Berkeley, CA.
- Hunt, J. & Andreasen, J. (2011). Making the most of Universal Design for Learning. *Mathematics Teaching in the Middle School*, 17(3), pp. 166-172.

Authors:

Dr. Cheng is an assistant professor in the Department of Mathematics at Towson University. She teaches mathematics education courses for pre-service and in-service teachers.

Ms. Horner is a secondary mathematics teacher pursuing her master’s degree in mathematics education from Towson University.

Challenging Students' Conceptions of Proportional Reasoning

Leigh Haltiwanger and Amber Simpson
Clemson University

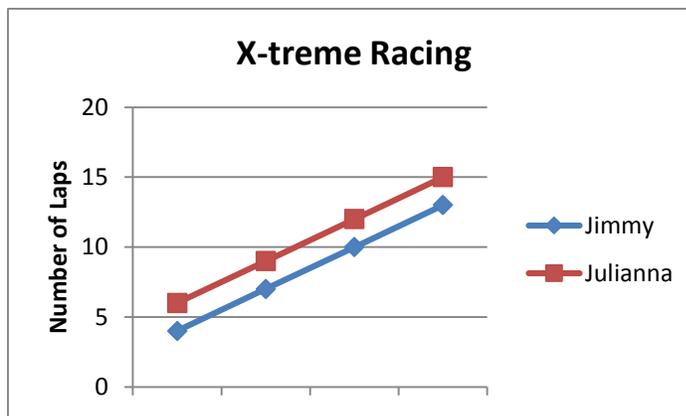
Abstract

Proportional reasoning is deemed as one of the most pivotal mathematical concepts for adolescents and is highlighted throughout the middle school mathematics curriculum in a variety of ways. This article focuses on developing students' proportional reasoning skills by considering differences between additive reasoning and multiplicative reasoning.

Proportional reasoning is deemed as one of the most pivotal mathematical concepts for adolescents (Hoffer & Hoffer, 1988; NCTM, 2000). It is highlighted throughout the middle school mathematics curriculum in a variety of ways. When students are learning about similarity, linear equations, probability, and volume, they should be developing a conceptual understanding of ratio and proportion (NCTM, 2000). As stated by the National Council of Teachers of Mathematics (NCTM) (1989), "the ability to reason proportionality develops in students throughout grades 5-8. It is of such great importance that it merits whatever time and effort that must be expended to assure its careful development" (p. 82). Moreover, the Common Core State Standards (CCSS) for Mathematics has devoted an entire strand, Ratios and Proportional Relationships, in the sixth and seventh grade standards (Council of Chief State School Officers, 2010). Also, a 2011 publication by Whitman, *It's All Connected: The Power of Proportional Reasoning to Understand Mathematics Concepts, Grades 6-8*, infuses the "big ideas" of proportional reasoning throughout both the NCTM Standards and the CCSS for mathematics. The book supports teachers as they "further help students see middle-school mathematics not as separate and unrelated concepts, but as concepts that revolve around a unifying idea or skill" (p. xii).

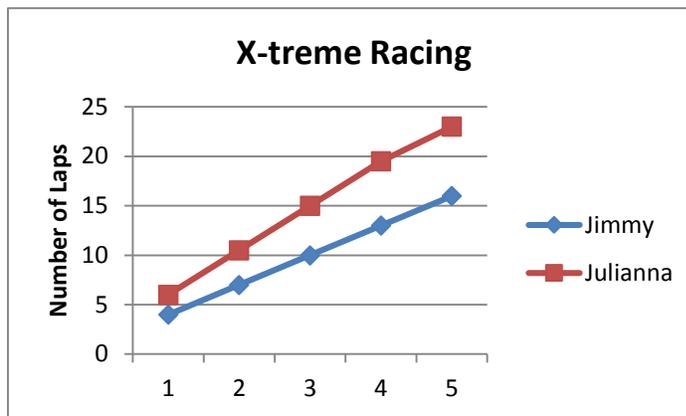
In this paper, we focus on developing students' proportional reasoning skills by considering differences that exist between additive reasoning and multiplicative reasoning. Additive reasoning, represented by a function of the form $f(x) = x + a$, is a single, constant quantity being added to an initial amount each time. On the other hand, multiplicative reasoning is represented by a function of the form $f(x) = bx$ and it compares change of a single quantity to an original value (Van de Walle & Lovin, 2006). For example, consider the following word problem: *Jimmy and Julianna are racing go-karts at the X-treme Racing Track. They drive equally as fast, but Jimmy started later. When Jimmy has driven 4 laps, Julianna has driven 6 laps. If Jimmy has driven 13 laps, how many laps has Julianna driven?* This problem can be solved using an additive method as implied by the statement "they drive equally as fast" and is represented here both as a table and as a graph. In this problem, the amount being added to Julianna's number of laps, as compared to Jimmy's number of laps, is 2. The function could be represented by $f(x) = x + 2$.

Jimmy	Julianna
4	6
7	9
10	12
13	15



Now consider the change in the wording of this problem: *Jimmy and Julianna are racing go-karts at the X-treme Racing Track. They begin racing at the same time, but Jimmy drives slower. When Jimmy has driven 4 laps, Julianna has driven 6 laps. If Jimmy has driven 13 laps, how many laps has Julianna driven?* The phrase *They drive equally as fast, but Jimmy started later*, in the previous problem was changed to *They begin racing at the same time, but Jimmy drives slower*, which indicates this problem is to be solved using a multiplicative method. In this problem, Julianna is driving 1.5 times faster than Jimmy. The function could be represented by $f(x) = 1.5x$. Again, we provide a tabular and graphical representation as a means to visually conceptual the problem.

Jimmy	Julianna
4	6
7	10½
10	15
13	19½



The transition from additive reasoning to multiplicative reasoning is difficult for students and in general, students incorrectly use additive methods to solve multiplicative situations and vice-versa. More specifically, students tend to overuse cross-products (i. e., $\frac{a}{b} = \frac{c}{d}$) to solve problems regardless of the situation (van Dooren, De Bock, & Verschaffel, 2010). Below we present two problems to encourage discussion and experimentation among students about the differences between the additive and multiplicative approaches. Before exploring the problems as a class, we also advise teachers to anticipate all the possible solutions their students may present (Smith & Stein, 2005). This will aid teachers in facilitating small group and whole group discussions, as well as making mathematical connections among students' arguments and explanations in the moment. Here are a few questions teachers might consider:

1. How would *you* approach these problems?
2. How would students in your classroom approach these problems?
3. What mathematics seems most important in these problems?
4. What challenging questions might you ask to students to extend their conceptual understanding?

The first problem is an informal activity that may serve well as a pre-assessment of students' current understanding of additive and multiplicative situations. The intent is not to obtain a "correct" answer, but to have students justify their choice through mathematical representations and appropriate use of language. The second problem will extend students' thinking by making connections between proportional reasoning and tabular and graphical representations, which may or may not be explored through technological tools (e.g., graphing calculator and Microsoft Excel). Although the cross-product method is a "go-to" method for students, this method is often used inappropriately (van Dooren et al., 2010) and may inhibit students' intuitive and conceptual understanding of proportional reasoning (Van de Wall & Lovin, 2006). Therefore, we would encourage students to approach these problems without using the cross-product method.

Problem 1: Running a Mile

This task, adapted from Van de Walle and Lovin (2006), encourages students to present three different arguments – one that would support each runner.

Max, Moe, and Minnie are required to run a mile as part of a fitness test in their gym class. They have recorded their time in minutes and seconds the first time they ran the mile in class (Week 0) and then at two-week intervals. After four weeks, which person has made the most progress in reducing their time? Provide a valid argument for all three runners – Max, Moe, and Minnie. Be prepared to defend your argument. As a note, 13 minutes and 4 seconds is represented by 00:13:04.

Week	Max	Moe	Minnie
0	00:12:11	00:11:34	00:10:58
2	00:12:04	00:11:28	00:10:51
4	00:11:58	00:11:22	00:10:46

Running a Mile Solution

To claim that Max made the most progress in reducing his time, students would utilize an additive approach. From Week 0 to Week 4, Max reduced his time the most, 13 seconds. Moe and Minnie, on the other hand, both reduced their time by 12 seconds over the four-week period. To defend this argument, students would subtract the time in Week 4 from Week 0 for each runner.

To defend Minnie as the most successful runner, students would employ multiplicative reasoning. Minnie lost the greatest percent of time (in seconds) from Week 0 to Week 4. Students would support this by taking the difference in time between Week 0 and Week 4 and divide by Minnie’s original time in seconds (Week 0), namely $\left(\frac{658-646}{658}\right) \approx 0.0182$ or 1.82%. Likewise, students would demonstrate that Moe had reduced his time by approximately 1.73%, $\left(\frac{694-682}{694}\right) \approx 0.0173$, and Max had reduced his time by approximately 1.78%, $\left(\frac{731-718}{731}\right) \approx 0.0178$.

For students to argue that Moe was the most successful runner, they would state that he reduced his time at a constant rate of change, 6 seconds every two weeks.

Extension Questions

- Assuming that the running times for Max and Minnie continue, after how many weeks will they be running a mile at the same time? Justify the solution with a table, with a graph, and algebraically.
- Why is it impossible to expect Moe to continue decreasing his running time at a constant rate? What about limiting the domain of the linear model representing Moe’s running time? Is this appropriate? Why or why not?

Problem 2: Plant Heights

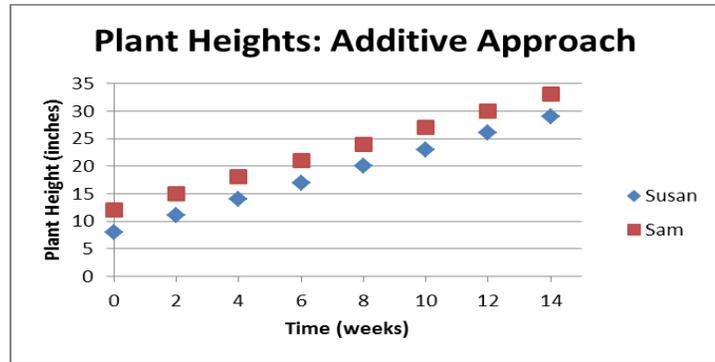
The following task, focusing on plant heights, is also adapted from Van de Walle and Lovin (2006). An accompanying handout is provided in Appendix A.

Two weeks ago, Susan and Sam planted flowers. Their plants were measured at 8 inches (Susan) and 12 inches (Sam), respectively. Today their plants are 11 inches (Susan) and 15 inches (Sam) tall. Did the 8-inch or 12-inch plant grow more? Provide an argument that Susan’s plant grew more and then provide an argument that Sam’s plant grew more.

Argument for Sam: An additive approach

The majority of students might first approach the problem from an additive approach, seeing that each plant increased their height at a constant rate of 3 inches over two weeks. While we want students to begin to recognize multiplicative approaches, this additive approach is also an appropriate way to approach this problem.

Time	Susan	Sam
0	8	12
2	11	15
4	14	18
6	17	21
8	20	24
10	23	27
12	26	30
14	29	33
...



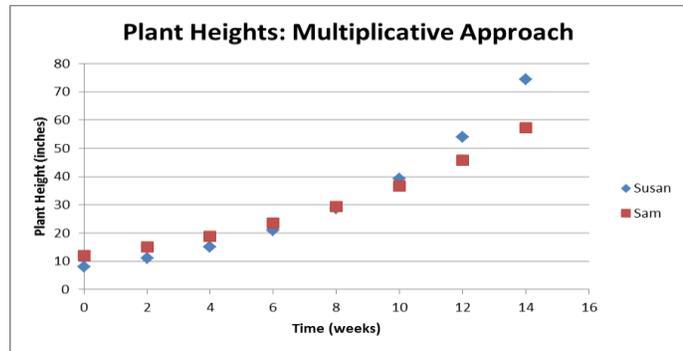
Note that in the table and graph, we began at $t=0$ to indicate the starting values given to us in the problem. We encourage having a conversation with students about this concept. Ideas about domain and range are critical and students should have time to explore thoughts about these features of the function. We've found it helpful, for example, to explicitly ask the students to think about their responses to the questions on the worksheet.

In generating a table and graphical display, students will notice that Susan and Sam's plants both have a constant rate of change; in other words, they both change by 3 inches every two weeks. This is illustrated in the table (adding 3 for each successive two week period of time) and on the graph by parallel lines. While the term arithmetic sequence may not be used, students should understand that this is an arithmetic sequence because the heights of the plants change by a constant value for every two weeks that pass. In other words, this function can be represented by $f(x) = x + 3$. Again, this argument is a correct approach to this problem and begins to lay the foundation for more complete understandings of intercepts and rates of change.

Argument for Susan: A multiplicative approach

To argue that Susan's plant has grown more, students will need to think about comparing the amount of growth of the plant after two weeks to the original height of the plant. Reasoning for this argument may be informal at first and be composed of comments such as, "If you look at the amount of growth for each plant, Susan's plant grows more than Sam's plant because $\frac{3}{8}$ is more than $\frac{1}{4}$." Specifically, students may notice that after two weeks ($t=2$), Susan's plant has grown at a faster rate than Sam's. (This argument is similar to Minnie's argument in the Running a Mile problem). This amount of growth can be expressed as a fraction: $(\text{new height} - \text{original height}) \div (\text{original height})$. In this case, Susan's plant's is growing at a rate of $\frac{3}{8}$ of its previous height $\left(\frac{11-8}{8}\right)$. Here, students may notice that at $t=2$, the new height can be obtained calculating $8 + \left(8 * \frac{3}{8}\right)$. In this formula, to obtain the new height, students will have to multiply the growth rate $\left(\frac{3}{8}\right)$, times the height of the plant before, and add that to the previous height of the plant $\left(\text{previous height} + \text{previous height} * \frac{3}{8}\right)$. So, for example, to find the height of Susan's plant after 4 weeks have passed, we would add the height of her plant at 2 weeks and add that to the growth rate of her plant during that time period $\left(11 + 11 * \frac{3}{8} = 15\frac{1}{8}\right)$.

Time	Susan	Sam
0	8	12
2	11	15
4	15.1	18.7
6	20.7	23.4
8	28.5	29.2
10	39.3	36.6
12	54.0	45.7
14	74.3	57.2
...



Students should follow a similar line of reasoning to find the height of Sam’s plant. Using multiplicative reasoning, students should recognize that Sam’s plant grows at a rate of $\frac{1}{4}$ of its previous height for each two week period that passes $\left(\frac{15-12}{12}\right)$. Again, to obtain the height of Sam’s plant after 4 weeks have passed, students will need to multiple the growth rate $\left(\frac{1}{4}\right)$ times the previous height of the plant (15) and add it to the previous height of the plant $\left(15 + 15 * \frac{1}{4} = 18\frac{3}{4}\right)$. If students extend this pattern in both a table and a graph, they will observe that Susan’s plant height exceeds Sam’s plant height after t=10 (10 weeks).

Again, while the phrase geometric sequence may not be used, students should recognize this as an argument that supports a constant multiplier to go from one height to the next. As in the additive argument for this problem, it is also critical to allow students to discuss how to represent information on the x-axis. For example, as before, we began with t=0. Students should have time to explore the meaning of this on the table and in the graph. Students should also be allowed to have adequate time to discuss the meanings of the intercepts, the domain and range of this problem, and the intersection point of the two functions.

While we understand that middle schools students’ mathematical reasoning is developing, we expect middle school students to recognize that Susan’s plant is growing at a faster rate than Sam’s if we consider the proportion of growth over time. We have found that helping students develop this multiplicative reasoning is vitally important. Challenging all students to begin to see multiplicative relationships prepares them for the algebraic concept rate of change.

Conclusion

Asking students to create additive and multiplicative arguments (as in the Plant Heights problem) helps them formalize ideas about ratio and proportional reasoning while, at the same time, engage them in several of the Standards of Mathematical Practice (CCSSO, 2010) such as making sense of problems and persevering in solving them (CCSS.Math.Practice.MP1) and constructing viable arguments and critiquing the reasoning of others (CCSS.Math.Practice.MP3). Furthermore, we believe that the discussion around the problem is more important than considering which argument is stronger or “right”. In the Plant Heights problem and in the Running a Mile problem, each person can have an acceptable argument for their plant height or increased running time. As stated by Hoffer and Hoffer (1988), “failure to develop in this area [proportional reasoning] by early to middle school adolescence preclude[s] study in a variety of disciplines requiring quantitative thinking and understandings” (p. 285). We believe that these two problems can serve as a beginning step towards helping students develop proportional reasoning skills.

3. Create a graph to represent your table.

4. How can you tell from looking at the table whose plant grew quicker? How can you tell from looking at the graph? In what ways are your arguments similar/different?

5. Compare your results with 1-4 with a partner. Do you agree or disagree and why?

Name _____ Date _____



***Plant
Heights,
Continued***



Directions: Work with a partner to create an argument that would help Susan *and* Sam claim that their plant grew quicker.

Susan's

Table:

Time	Susan	Sam

Graph:

Argument:

6. What do you think can be the maximum height of Susan's plant? Why?
7. What do you think can be the minimum height of Susan's plant? Why?
8. How tall will Susan's plant be after 18 weeks? How do you know? Do you think that this is realistic?

Sam's Argument

Table:

Time	Susan	Sam

Graph:

Argument:

9. What do you think can be the maximum height of Sam's plant? Why?

10. What do you think can be the minimum height of Sam's plant? Why?

11. How tall will Sam's plant be after 18 weeks? How do you know? Do you think that this is realistic?

Appendix B: Additional Problems

1. Two friends mix blue tint with white paint to make some blue paint. Decide which friend mixed the darkest shade of blue paint. Nancy used more blue tint than Kathy. Nancy mixed in more white paint than Kathy. Who mixed the darkest shade of blue?

- A. Nancy
- B. Kathy
- C. Their paint mixtures were exactly the same.
- D. There is not enough information to tell.*

Thompson, C. S., & Bush, W. S. (2003). Improving middle school teachers' reasoning about proportional reasoning. *Mathematics Teaching in the Middle School*, 8(8), 398-403.

2. Josiah the snake is 3 feet long. When he is fully grown, he will be 6 feet long. Judy the snake is 5 feet long. When she is fully grown, she will be 8 feet long. Which snake is closer to being fully grown? Explain how you know.

Adapted from: Thompson, C. S., & Bush, W. S. (2003). Improving middle school teachers' reasoning about proportional reasoning. *Mathematics Teaching in the Middle School*, 8(8), 398-403.

3. An onion soup recipe for 8 people:

- 8 onions
- 2 cups of water
- 4 chicken soup cubes
- 12 T. butter
- $\frac{1}{2}$ cup cream.

- A. I am cooking onion soup for 4 people. How many chicken soup cubes do I need? Show your work both algebraically and through a pictorial representation.
- B. I am cooking onion soup for 6 people. How much cream will I need? Show your work both algebraically and through a pictorial representation.

Rutchi, W. P., & Bennett, C. A. (2013). Develop reasoning through pictorial representation. *Mathematics Teaching the Middle School*, 19(1), 30-36.

4. Which figure has more circles?

Figure A



Figure B



Van de Walle, J. A., & Lovin, L. H. (2006). *Teaching student-centered mathematics: Grades 5-8*. Boston, MA: Pearson.

References

- Council of Chief State School Officers. (2010). *Common Core State Standards for Mathematics*. Washington D.C.: National Governors Association Center for Best Practices, Council of Chief State School Officers.
- Goerdt, S. & Horton, B. (2011). *Fostering Algebraic Thinking with Casio Technology*. Dover, NJ: Casio.
- Hoffer, A., & Hoffer, S. (1988). Ratios and proportional thinking. In T. Post (Ed.), *Teaching mathematics in grades K-8* (pp. 285-312). Boston, MA: Allyn & Bacon.
- National Council of Teachers of Mathematics (NCTM). (1989). *Curriculum and Evaluation Standards for School Mathematics*. Reston, VA: NCTM.
- NCTM. (2000). *Principles and standards for school mathematics*. Reston, VA: NCTM.
- Smith, M. S., & Smith, M. K. (2005). *5 practices for orchestrating productive mathematics discussions*. Reston, VA: NCTM.
- Van de Walle, J. A., & Lovin, L. H. (2006). *Teaching Student-Centered Mathematics: Grades 5-8*. Boston: Pearson.
- Van Dooren, W., de Bock, D., Verschaffel, L. (2010). From addition to multiplication...and back: The development of students' additive and multiplicative reasoning skills. *Cognition and Instruction*, 28(3), 360-381. doi: 10.1080/07370008.2010.488306
- Whitman, C. (2011). *It's all connected: The power of proportional reasoning to understand mathematics concepts, grades 6-8* (xi-xvi). Retrieved from http://www.mathsolutions.com/documents/9781935099246_AboutResource.pdf

Authors:

Leigh Haltiwanger teaches in the Eugene T. Moore School of Education at Clemson University and is also working on her PhD in Curriculum & Instruction: Mathematics Education.

Amber Simpson is an advanced doctoral student at Clemson University, pursuing a degree in Curriculum & Instruction: Mathematics Education.

A Technology Makeover for the Glyph

Pamela D. Wash
Winthrop University

Elizabeth L. Syracuse
University of South Carolina Upstate

Abstract

“If we teach today as we taught yesterday, we rob our children of tomorrow” (Dewey, 1916). This statement, made almost one hundred years ago, is even more relevant in today’s classrooms with the push to seamlessly integrate technology. K-12 schools are responding to this demand by designing a curriculum in which students learn and show proficiency through 1:1 technology initiatives rather than a curriculum in which technology is simply an available tool. To that end, teachers are searching for ways to create lessons that address content standards while incorporating and taking advantage of technology. Glyph lessons that have received a technology makeover are a time efficient way of teaching varied mathematical skills, including data analysis in the context of technology.

Background

With the surge of K-12 classrooms moving to 1:1 computing where every student is provided a mobile device and “Bring Your Own Device” (BYOD) initiatives, it is imperative that classroom teachers take full advantage of the technology literally right at the students’ fingertips to move teaching and learning into the 21st century. One way to do this is to *makeover* standards-based lessons to meet both the appropriate content standards as well as the International Society for Technology in Education Standards for Students (ISTE-S) (2007).

“1:1 computing may be a goal at many schools across the country, but it’s a way of life at 22 elementary campuses in the Texas Panhandle” (Solomon, 2006, pg. 26). This is also evident in South Carolina where one of the largest school districts, Richland School District 2, implemented a 1:1 computing initiative beginning in 2011 with gradual implementation over a two-year period. The target for this initiative was to outfit over 19,000 students in grades 3-12 with committee selected *Google Chromebooks* by August 2013 (District Administration Custom Publishing Group, 2012). Spartanburg School District 7, also a school district in South Carolina with over 7,100 students, launched a 1:1 computing initiative in the fall of 2013 entitled *Seven Ignites* (Spartanburg School District Seven, 2013). Students in grades 3-5 received iPads, students in grades 6-12 received MacBook Air laptops and each teacher received both an iPad and a MacBook Air. As monies allow, more and more school districts are moving toward this format of instructional technology classroom implementation.

“Since 2009, the number of teens with a cell phone has risen dramatically, with 58% of students at age 12 having a cell phone and 83% of 17-year olds owning a cellular device”, (Sutton, 2013, pg. 34). With this demographic knowledge, some schools are electing to use a Bring Your Own Device (BYOD) initiative. As a result, schools and teachers are asking students to bring any Internet accessible device like tablets, e-readers, and cell phones to the classroom for use during instruction. While this approach seems to be gaining popularity in colleges more so than in K-12 settings, schools and classroom teachers are making the choice to use this approach when school-wide devices are not readily accessible.

“Educational technology is a student motivator,” according to the 2013 survey results released by PBS LearningMedia. Results from the survey administered to 500 K-12 teachers revealed that 74% believe that technology expands content and motivates students to learn, and 73% believe that technology allows teachers to meet the needs of a variety of learning styles. The survey also revealed that more than two-thirds of the teachers responded that they yearn for more technology in their classrooms. While the data are not clear on how either of these two computing initiatives are increasing academic performance in K-12 classrooms, it is clear that it motivates students and increases accessibility to the most current content and tools (Ramig, 2014).

Glyph Lesson Makeover

Teaching is an ever changing and morphing profession. With the adoption of the Common Core State Standards (NGA Center & CCSSO, 2010) coupled with existing emphasis of the ISTE Standards (2007) and value-added assessments, classroom teachers are scrambling to update, revise, and design content lessons to uncover and integrate these student academic expectations. Therefore, it is critical that teachers conduct lesson makeovers to meet these evolving instructional demands.

Do you remember using a glyph in your classroom? A glyph is a pictorial representation of data that can be used to generate graphs and other data analyses engagements. According to Magnuson (2010, pg. 1), “the construction and interpretation of glyphs that represent information about themselves and other topics is an interesting, useful, and engaging math lesson!” Glyphs can be designed to include many mathematical skills such as: reading keys, organizing statistical data, number recognition, counting, identifying and/or drawing geometric shapes, making patterns, comparing and contrasting, positional and spatial relationships, and thinking, reasoning, and communicating mathematically (Magnuson, 2010). So how can we take what was typically a paper-pencil, color, cut and glue lesson in the classroom and move it to a 21st century classroom math lesson? Simply redesigning the concept focusing on available technologies will move this lesson to a more contemporary lesson for today’s classrooms.

Tech Update

A typical classroom glyph student sheet includes a legend containing attributes and pieces or components that students assemble according to the legend to make a pictorial representation of their personal attributes. In the past, these components have been hand-colored, cut out, and glued for display. With the accessibility of technology in today’s classrooms, the student sheet for the glyph can be created electronically; therefore, manipulated by the student electronically (see Appendix A). Images, used from Creative Commons sources, can be provided in a Microsoft Word document and the document disseminated to students via a content management system or free online file sharing tools like Padlet (<http://padlet.com/>), Google Docs (<https://docs.google.com/>), or OneDrive (<https://onedrive.live.com/about/en-us/>). Students can then easily access the file, drag and drop the images and components as needed to assemble their glyph, save, and share their final products via printing or file sharing.

Graphing Extension Update

Once all glyphs are created and shared with the class, students can then use their laptops or tablets to generate graphs of the results. Questions and classroom discourse regarding the data results can be conducted, i.e. How many more students have feet of length equal to or greater than 6 ½ inches than students with feet less than 6 ½ inches? One student-friendly, free online graphing tool to consider, *Create a Graph* (<http://nces.ed.gov/nceskids/createagraph/default.aspx>), is published and provided by the National Center for Education Statistics. This program allows the user to select the most appropriate type of graph, including 2D and 3D options, customize the data appearance as well as the data labels, plot area, scale, etc. After creating a graph, users can save a provided hyperlink to access the graph later for editing, can download and save a copy as a PDF or JPG file, or simply print out a copy (see Appendix B). Two additional online graphing tools to consider include *RapidTables* (<http://rapidtables.com/tools/bar-graph.htm>) and *ChartGo* (<http://www.chartgo.com/>).

Updated Glyph Example

The following glyph makeover example integrates specific skills from the CCSS (2010) for Mathematics Third Grade *Measurement and Data: B3. - Draw a scaled picture graph and a scaled bar graph to represent a data set with several categories. Solve one- and two-step "how many more" and "how many less" problems using information presented in scaled bar graphs. B4. - Generate measurement data by measuring lengths using rulers marked with halves and fourths of an inch. Show the data by making a line plot, where the horizontal scale is marked off in appropriate units - whole numbers, halves, or quarters* and ISTE-S Standards 2. *Communication and Collaboration*; 5. *Digital Citizenship*; and 6. *Technology Operations and Concepts*. By taking time to make technology updates to existing or original lessons, teachers can reinforce content standards, seamlessly integrate ISTE-S standards (2007) as well as motivate students and meet varying learning styles. To download a classroom ready version of the following glyph, please visit <http://padlet.com/pwash/ybrzdt1a9vha>.

Apple Glyph



A glyph is a form of picture writing that conveys information and data that can be analyzed and graphed.

Directions:

- Create your glyph following the legend below, the “parts” provided on the following two pages, and a ruler.
- Using the provided ruler, determine which attribute matches your characteristics, then simply drag and drop the appropriate parts to assemble your glyph. Please delete any unused parts by clicking on each unused part and clicking the delete button.
- When your glyph is complete, do a “save as”, print it out and place it on the board up front.
- After all glyphs are posted, use one of the following online graphing programs to create a **bar graph** representing our class’ data.
 - *Create a Graph* (<http://nces.ed.gov/nceskids/createagraph/default.aspx>)
 - *RapidTables* (<http://rapidtables.com/tools/bar-graph.htm>)
 - *ChartGo* (<http://www.chartgo.com/>)
- When your graph is complete, download your graph as a PDF and print it to be turned in.

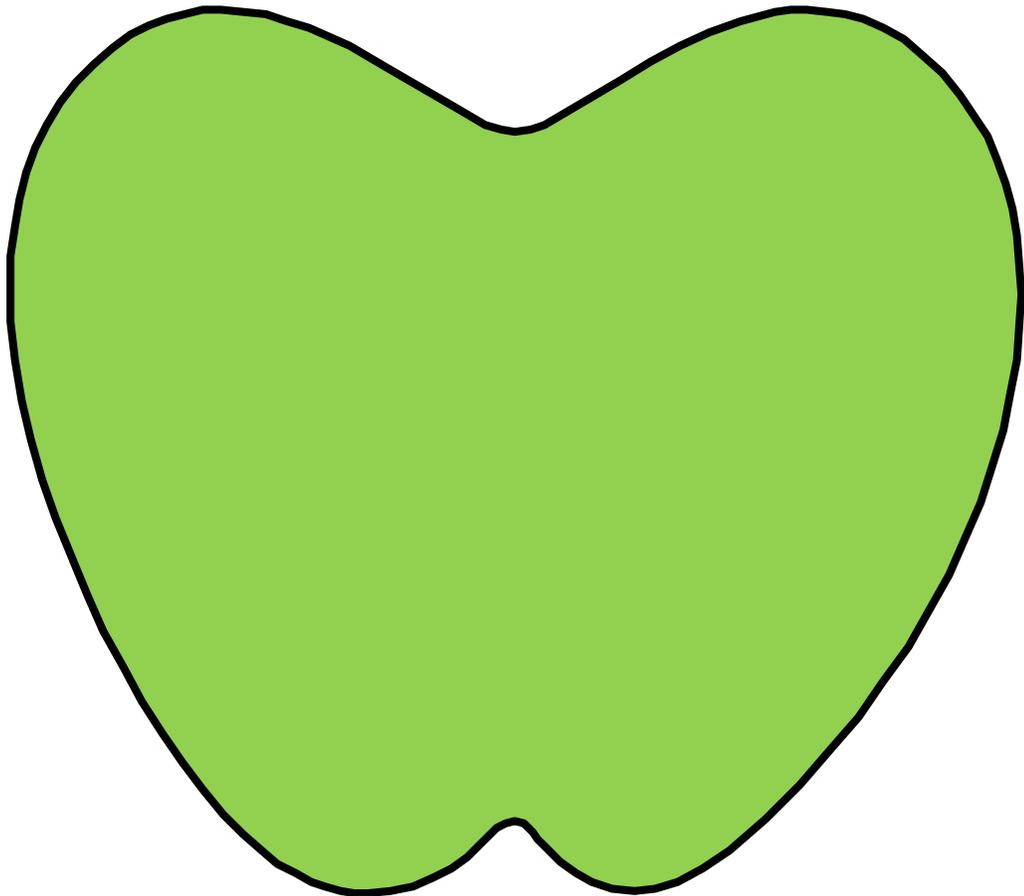
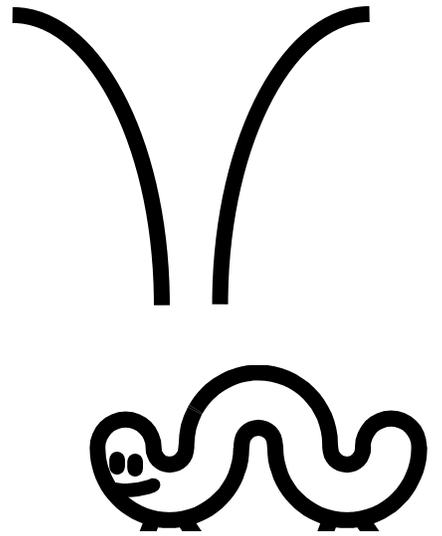
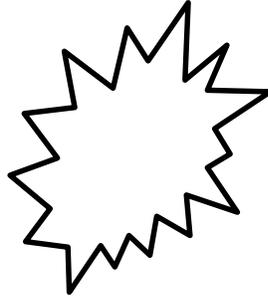
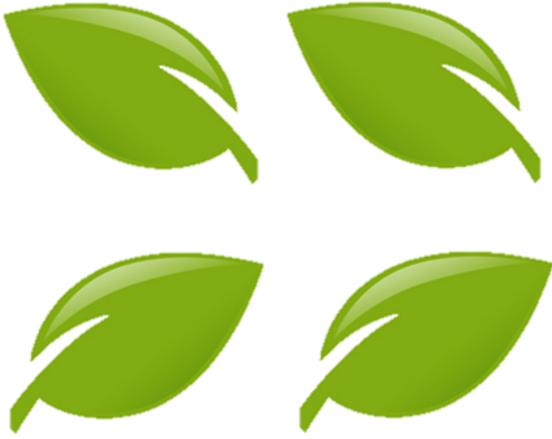
Construct your glyph using the following legend:

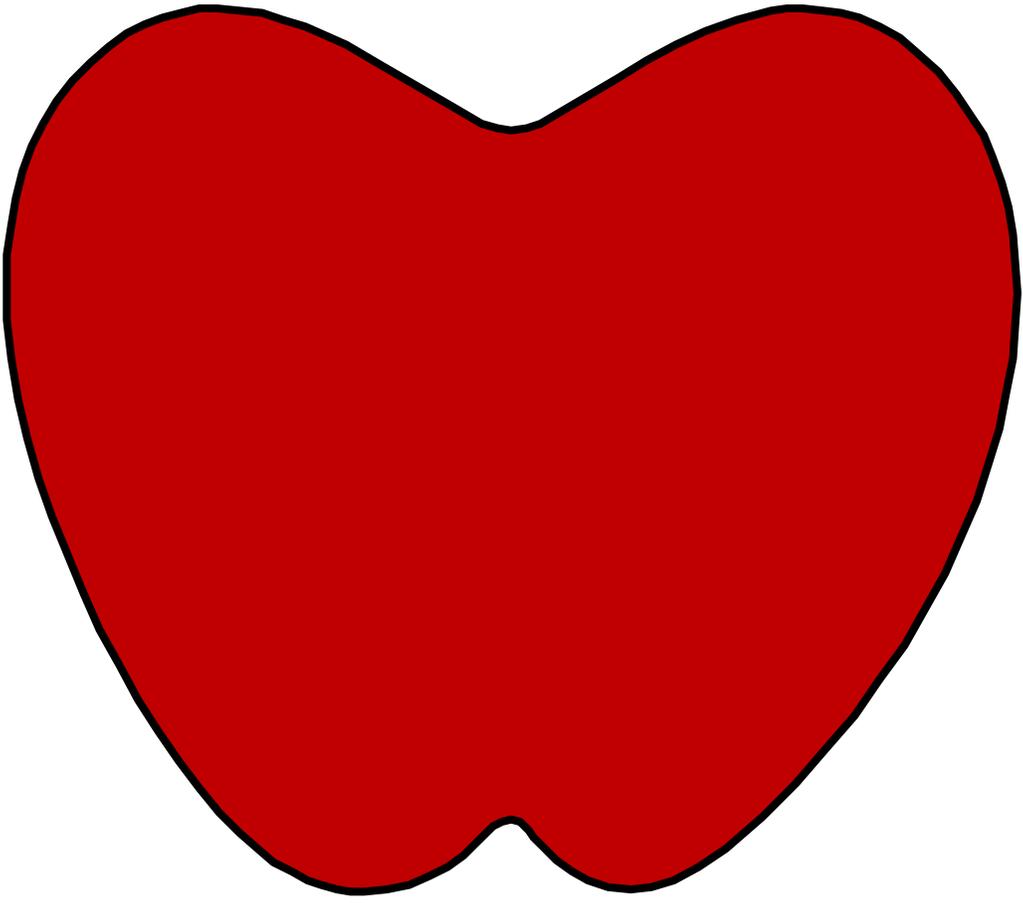
LEGEND

Item	Attribute Meaning
Apple	<ul style="list-style-type: none"> • Green Apple if your foot is greater than or equal to 6 ½ inches • Red Apple if your foot is less than 6 ½ inches
Stem	<ul style="list-style-type: none"> • Stem Pointed Right if your pointer (index) finger is greater than or equal to 2 ¼ inches • Stem Pointed Left if your pointer (index) finger is less than 2 ¼ inches
Worm	<ul style="list-style-type: none"> • Worm if the distance from the bottom of your knee cap (patella) to your ankle (tarsus) is greater than or equal to 5 ½ inches • No Worm if the distance from the bottom of your knee cap (patella) to your ankle (tarsus) is less than 5 ½ inches
Leaves	<ul style="list-style-type: none"> • Three Leaves if your hand span (thumb to pinky outstretched) is greater than or equal to 4 ½ inches • One Leaf if your hand span (thumb to pinky outstretched) is less than 4 ½ inches
Bite Mark	<ul style="list-style-type: none"> • No Bite Mark if your humerus (the distance from your shoulder to your elbow) is greater than or equal to 5 ¼ inches. • Bite Mark if your humerus (the distance from your shoulder to your elbow) is less than 5 ¼ inches

My Apple Glyph

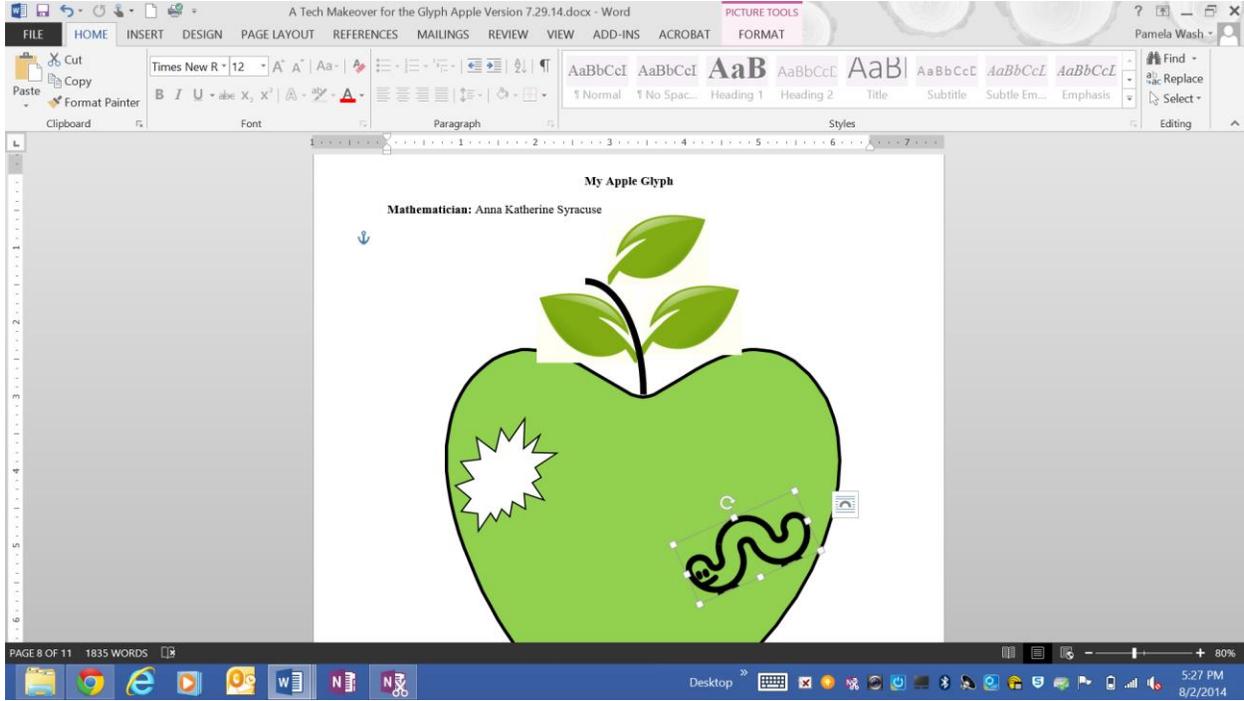
Mathematician: (insert your name here)





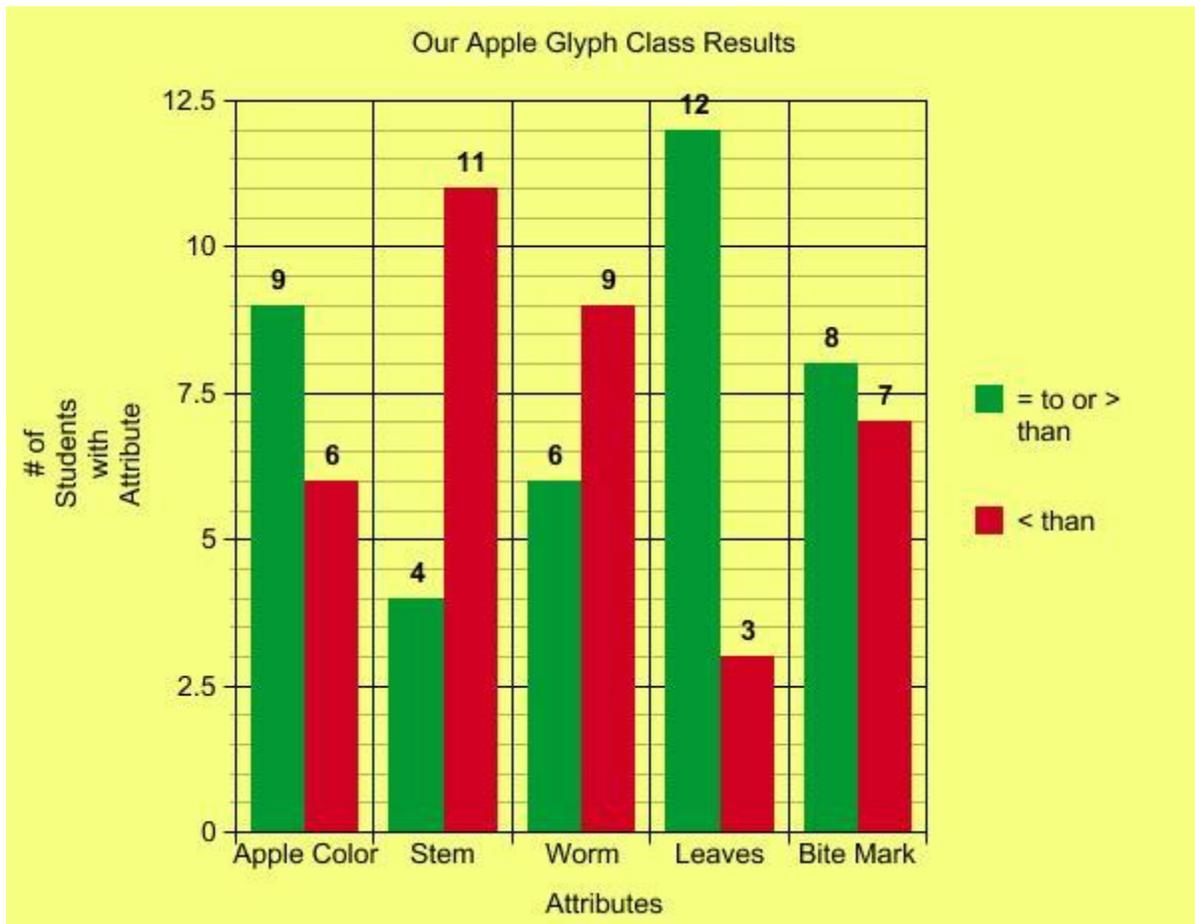
Appendix A

Glyph Manipulation Screen Shot



Appendix B

Graph Example



References

- Dewey, J. (1916). *Democracy and education: An introduction to the philosophy of education*. NY: Freepress
- District Administration Custom Publishing Group. (2012). Large s.c. district rolls out 1:1 computing initiative: Richland school district two administrators choose google chromebooks for thousands of students. *District Administration, July/August 2012*.
- ISTE Standards – Students. (2007). *International Society for Technology in Education*. Retrieved from <http://www.iste.org/standards/standards-for-students>
- Magnuson, A. (2010). Data gleaning from glyphs: Math in pictures for kindergartners. Retrieved from <http://www.scholastic.com/teachers/classroom-solutions/2010/09/data-gleaning-glyphs>
- National Governors Association Center for Best Practice & Council of Chief State School Officers. (2010). *Common Core State Standards for Mathematics*. Washington, D.C.: Author.
- Public Broadcasting Service LearningMedia. (2012). PBS survey finds teachers are embracing digital resources to propel student learning. Retrieved from <http://www.pbs.org/about/news/archive/2013/teacher-tech-survey/>
- Ramig, R. (2014). One-to-One computing and learning: Has it lived up to its expectations? *Internet@Schools, 21*(2), 6-8.
- Solomon, J.D. (2006). 1:1 computing at the elementary level in texas: Handheld program for reading and math includes 3,200 third-through fifth-grade students in amarillo. *District Administration, June 2006*.
- Spartanburg School District Seven. (2013). Seven ignites. Retrieved from <http://7ignites.com/>
- Sutton, S. L. (2013). STEM Signals. *Techniques: Connecting Education & Careers, 88*(6), 34-37.

Authors:

Pamela D. Wash is an associate professor in the College of Education at Winthrop University and serves as the Department Chair of Counseling.

Elizabeth L. Syracuse is a Master of Education in Elementary Education candidate at the University of South Carolina Upstate. She is a certified special education teacher who now resides in Germany.