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THE MATHMATE

THE OFFICIAL JOURNAL OF THE
SOUTH CAROLINA COUNCIL OF TEACHERS OF MATHEMATICS

Mission Statement: The mission of THE MATHMATE is to feature articles about innovative mathematical classroom practices, important and timely educational issues, pedagogical methods, theoretical findings, significant mathematical ideas, and hands-on classroom activities and disseminate this information to students, educators and administrators.

THE MATHMATE, the official journal of the South Carolina Council of Teachers of Mathematics, is published online three times each year – January, May, and September.

Submission Requirements: Submissions for THE MATHMATE should be no more than 15 pages in length not counting cover page, abstract, references, tables, and figures. Submissions of more than 15 pages will be reviewed at the discretion of the editorial board. Submissions should conform to the style specified in the *Publications Manual of the American Psychological Association* (6th ed.). All submissions are to be emailed to scmathmate@gmail.com as attachments with a completed Submission Coversheet as page 1 and the article starting on page 2. Click here to download THE MATHMATE Submission Coversheet.

Submitted files must be saved as MSWord, RTF, or PDF files. Pictures and diagrams must be saved as separate files and appropriately labeled according to APA style. Copyright information will be sent once an article is reviewed but authors should not submit the same article to another publication while it is in review for THE MATHMATE.

Submission Deadlines: Submissions received by October 1 will be considered for the January issue, February 1 for the May issue, and June 1 for the September issue.

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The MathMate

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Message from the SCCTM President

Dear Members,

The board of SCCTM would like to thank all the authors to this MathMate as well as to the editor Gina Dunn for making this edition possible. The MathMate was and still is an important part of our organization. It provides our members with excellent examples of best practices as well as providing classroom activities that we all can use. I hope you enjoy this edition and will think about contributing your expertise in future editions. Be sure to attend the conference on November 10^{th} and 12^{th} to share and learn new mathematics strategies that will help the children of South Carolina become better mathematicians. Again let me thank Gina and all those who helped her put this edition out.

Bill Whitmire
2015-2016 SCCTM President

Announcements

Upcoming Conference Information and Deadlines:

SCCTM Fall Conference
SCCTM 2016 Annual Fall Conference, Greenville, South Carolina,
November 10 - 11

Membership News:

Renew your NCTM membership online and designate *South Carolina Council of Teachers of Mathematics* for the affiliate rebate.

If you would like your announcement to appear in the next issue of THE MATHMATE, please email all information to SCMathMate@gmail.com by January 1, 2017. Announcements will be published at the discretion of THE MATHMATE Editorial Board.

Guest Editorial

P3--Professionalism, Publishing, and Practice

Deborah Bass

Teachers always have a thirst for learning. I found that the South Carolina Council of Teachers of Mathematics offered an avenue to quench mine with resources for my learning, as well as with ways to contribute to others' learning. My introduction to the SCCTM was first through *MathMate* when I was practice teaching in a middle school. I had completed a master's degree (in journalism) several years earlier, but now my interest turned toward teacher certification which eventually required another master's degree to fulfill certification requirements.

When I made it to the point of student teaching, my cooperating teacher could not begin the year because of health reasons until six weeks into the year. Consequently, I started the year teaching all her math classes. By that sixth week in algebra, I was glad she made it back! During these early teaching experiences, I wanted to be creative in a way that would engage the students and challenge them to see math in all aspects of life. Their struggles indicated to me that math was only in a textbook for them, without any relevance to their world or way of life.

I began devising ways for students to work problems and make math connections to other content areas. One finished product I designed was a geography trivia game with questions and answers that fit into a mathematical equation. The formula included many of the skills the students were practicing and it was long enough to maintain interest, a bit of competition, and collaboration. My cooperating teacher, who was truly a math expert, worked with me to ensure accuracy of the formula. All went well and afterwards, my cooperating teacher suggested that I submit the activity to *MathMate*. The activity was published and I knew that SCCTM was really for teachers like me. *MathMate* was a channel for teachers to share with and learn from each other and I really liked that!

From that introduction to the SCCTM, my interest in membership was certain because I wanted to stay connected to other professionals in the field of mathematics who were willing to share their knowledge. I enjoyed reading *MathMate* to find content relevant to my teaching.

Not only did SCCTM offer a journal for educators, but also a conference! Often in my public school teaching career, debate would surface when leaving the classroom for a day or so to attend a conference. But, I believe the value of a conference is found ultimately back in the classroom. The SCCTM was inclusive of all professionals in math and no matter which level I taught, there was something for me to gain a deeper understanding of the current topics or techniques.

Staying abreast of the broader implications of classroom teaching and learning always enriched my teaching and helped me make connections in the profession in the same manner that I wanted my students to do in the classroom. I found it rewarding to gain insight from colleagues whose practices and research validated my decisions or encouraged me to make some changes to develop the broader aspects of my work. That's what SCCTM membership offered me.

I enjoyed *MathMate* and the conferences, and at the first opportunity, I became a lifetime member to ensure that I would kept track of current trends in math education.

In my teaching and in my conversations with parents, I heard about the difficulties posed by math. There was often a search for how to make math more manageable or easier. My staying in touch with others who studied and practiced teaching math helped me with these conversations. Through the SCCTM, I met other professionals who were eager to discuss their techniques.

When I taught mathematics methods at the University of South Carolina Aiken, I knew teacher preparation had to include ways to connect and understand math in everyday life. One assignment I enjoyed was for students to develop a math project that would make a real-life connection. These pre-service teachers always presented wonderful information about how math was used in professions and activities from roofing, to golf, to cutting glass for windows. Their work could easily be duplicated at any grade level. Years later, as a principal, I observed a former student using this same project with her students. This reminded me of how through the SCCTM teachers could share with teachers and learn from each other.

SCCTM provided an opportunity for educators to connect with fellow researchers and classroom practitioners. Ultimately, I encouraged my students to make connections and search for ways to develop a rich dialogue about the profession in their content area.

I have enjoyed every level of teaching in my career from the elementary, to middle, to high school classrooms, as well as teaching and working with pre-service and in-service teachers. No matter the level, sharing information on how to become better at teaching, particularly in mathematics, is as beneficial to students as it is to the teachers. In my career, when I have learned more personally, I have shared more professionally.

Needless to say, my introduction to professional education publications and organizations began with the SCCTM. I am delighted South Carolina educators have established a way to connect professionals in mathematics. The organization gave me a start in publishing my work, as well as a source for ideas to teach mathematics. As I implemented teaching math with literature, manipulatives, writing and making real life math connections, I saw that others shared these same type ideas when I visited conferences and read professional journals. I hope others will begin and continue, as I did, with mathematics and the SCCTM. It certainly serves all levels in the educational community, as I found during my teaching career.

Modeling Figure Skating Upright Spins:
A Geometry & Trigonometry Activity

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Towson University

Abstract

We provide secondary-level mathematical modeling activities based on the motion of the arms during a figure skating upright spin. The activities in this article are based on video recordings and dynamic geometry representations of a skater performing this spin. Students must apply their mathematical knowledge to the real-life scenario in order to solve the problems posed. Using the Geometer's Sketchpad animation, they apply concepts of triangular geometry and fundamental trigonometry to investigate the motion of a figure skater's arms during an upright spin.
Keywords: Modeling, Figure Skating, Geometry, Trigonometry

In 2015, eleven-year old Canadian figure skater Olivia Rybicka-Oliver became the fastest spinner on the ice, rotating at a rate of 342 revolutions per minute (Guinness Book of World Records, 2015). The spin that Rybicka-Oliver performed was an upright spin. In this article, we expand upon a previously published activity (Authors, 2016) with a more complex model of the upright spin and in-depth mathematical questions related to this model. While working with an animated model, students are expected to improve their knowledge of sine and cosine laws, polygon and circular geometry and mathematical modelling. Mathematical modeling involves linking mathematics and questions which have been posed about real-world situations (Cirillo et. al., 2016).

A model of the arms in the upright spin

The upright spin is the most basic type of spin that figure skaters perform. The skater begins on a curved path but then rotates counterclockwise around a fixed vertical axis established by her skating leg. If the skater wishes to increase angular speed within the spin, the distance of body parts from this vertical axis must be reduced (Petkevich, 1989). In order to complete an upright spin properly, skaters must be aware of the exact position of each body parts through the entire duration of this figure skating element.

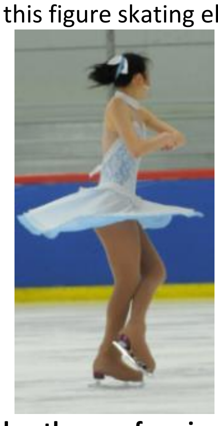


Figure 1. Second author performing an upright spin.

In our activity, we focus on the motion of the arms during the spin, since different positions of the arms impact a skater's balance and angular velocity. At some moments in the spin, the skater extends the arms out and then pulls them in towards the torso. By changing the position of the arms, the skater increases and decreases the radius of the spin, controlling the change of angular velocity of the body (King, 2011).

This activity is designed to investigate geometry and trigonometry associated with dynamic positions of the arms from full extension to when the skater bends his or her elbows to bring the forearms in towards the torso. Since a skater's upper and lower arms have fixed lengths, their trajectories are sectors of circles. When the elbows bend, they become closer to the skater's torso. We assume that the left and right sides of the body are moving symmetrically and simultaneously, at a constant rate, throughout this stage of the spin. To meet the needs of the targeted audience, we ignore the additional distance traveled by the rotation of the skater's body.

Task Set-Up

We suggest that teachers begin this activity by showing a video of a skater demonstrating an upright spin. A video that corresponds with the model portrayed in this activity is available here: http://www.youtube.com/watch?v=xTKAc-ukXSc**.** In this video, the second author demonstrates an upright spin so that students can see how all of the body parts move during this figure skating element; she then stops spinning and isolates the motion of the arms in a way that corresponds to this activity. For the purposes of creating a mathematical model accessible to high school students, we ignore the circular rotation of the skater when discussing arm motion.

After watching the video, teachers should ask students to imagine what the arms' motion would look like if another video were taken from directly above the skater, see Table 1. While we do provide a model of the arms in the activity, we suggest that students be first given the opportunity to create this model themselves. In doing so, students can experience translating a problem into a mathematical form, a practice which Cirillo et. al. (2016) consider to be the "essence" of mathematical modeling (p. 5). As a way of improving students' understanding of the arms during the upright spin, teachers could discuss what shapes are being formed by the arms, as well as why skaters might form these shapes with their arms. The activity sheets can be completed without accessing our accompanying Geometer's Sketchpad animation, but the animation could be used to enhance students' understanding of the model. **From modeling to real-life interpretations**

Providing situations in which students must represent real-life scenarios, examine mathematics within these representations, and interpret the mathematics is fundamental to mathematical modeling (CCSSI, 2010). Another discussion that teachers could have is to compare the arm motion models provided in this activity and the activity published in (Authors, 2016). The model provided in this activity is more authentic and closer to how skaters normally move their arms. The reason is that by moving the elbows (rather than holding them in a fixed location, as shown in Authors, 2016), skaters are able to decrease the distance of their furthest body part from their axis of rotation, and thus skaters can rotate faster in the tighter position. The ideas behind this upright spin activity could be further extended to include the physics of angular momentum (Lewin, 2009).

Extensions of this exploration

One way to extend this exploration is to include a discussion of the physical principle of conservation of angular momentum, which applies in the upright spin (Lewin, 2009). The skater's arm motions - from extended away from the skater's body to pull closer to the skater's body – are completed because the skater wants to decrease the radius (the distance of the body part furthest from the skater's center of rotation) in order to increase angular velocity. In fact, the skater's angular velocity is inversely proportional to the square of the radius. Teachers could ask students to recommend skaters' positions of the arms and elbows if the skater wishes to increase angular velocity by certain proportions.

Another way to extend this exploration is by having students develop more questions based on the existing representation of the bird's eye view of the arms. The Figures in this activity were created using a Geometer's Sketchpad animation that we have made accessible at the following website: https://sites.google.com/site/mathematicswithinanuprightspin/home.

Students can use the "Animate" button to observe how the elbow line moves during the spins, how the arms move from an extended position to the pulled in position, and to "trace" the trajectories that the elbows and hands travel.

If teachers would like to create different figures for the purposes of assessment or enrichment, they may use this file to change the lengths of the various line segments. Dragging point H along the vertical line of reflection will proportionally resize the entire diagram.

Some additional questions that teachers could ask students to explore using the animation include: 1) Assuming that the center of rotation is at the skater's head, how far is the furthest point of the skater's body from the center of rotation?; 2) How far does the right hand move during the animation; and 3) If the skater rotates 10 times during the spin, how far does the right hand actually move (taking into account the skater's rotation)?

Other open-ended questions related to the context that could be posed include: What would happen if the skater's arms did not move symmetrically? What would happen if the skater's arm length was longer or shorter? How would the model change if the skater twists his or her torso?

Common Core State Standards addressed

The questions posed in the activity address the following Content Standards (CCSSI, 2010):

- HSG.MG.A.1: Use geometric shapes, their measures, and their properties to describe objects (e.g., modeling a tree trunk or a human torso as a cylinder).*
- HSG.GMD.B.4: Identify the shapes of two-dimensional cross-sections of three-dimensional objects, and identify three-dimensional objects generated by rotations of two-dimensional objects.
- HSG.SRT.D.11: Understand and apply the Law of Sines and the Law of Cosines to find unknown measurements in right and non-right triangles (e.g., surveying problems, resultant forces).
- HSG.C.B.5: Derive using similarity the fact that the length of the arc intercepted by an angle is proportional to the radius, and define the radian measure of the angle as the constant of proportionality; derive the formula for the area of a sector.

Activity Sheet: Geometry & Trigonometry in an Upright Spin

In this activity, we explore mathematics of the arms' motions from the bird's eye view. On the right hand column of Figure 1, we illustrate the bird's eye view of three positions of the upper arms during the spin, constructed using dynamic geometry software.

The following points represent the skater's body parts as listed below:

- Point A: skater's right shoulder
- Point S: initial position of skater's right elbow
- Point E: dynamic position of skater's right elbow
- Point M: initial position of skater's right hand
- Point R: dynamic position of skater's right hand
- Point S': initial position of skater's left elbow
- Point D: dynamic position of skater's left elbow
- Point M': initial position of skater's left hand
- Point L: dynamic position of skater's left hand

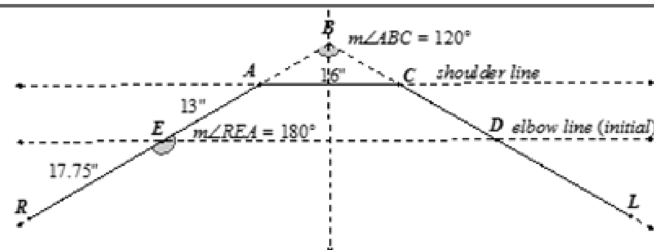
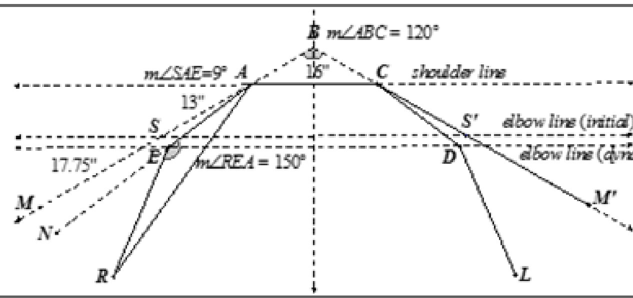
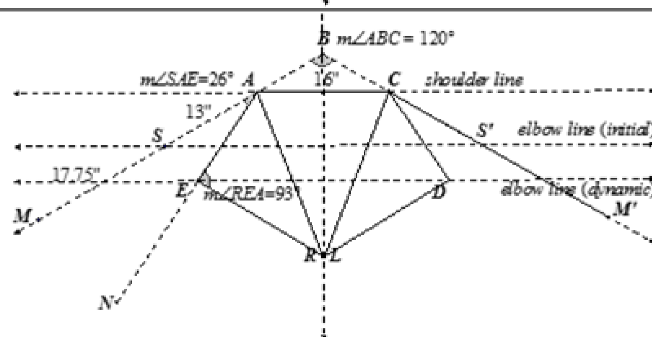
Time (sec)	Description of arms	Front View (body action)	Bird's Eye view of Arms
0	Extended		
3.93	Bent at shoulders and elbows		
12.7	Hands just touched		

Table 1. Upright spin – model of arms from front and side views at three time intervals.

Refer to Table 1 and answer the following questions regarding the arms' motions in the upright spin. For all trigonometric calculations, round to the nearest ten-thousandth.

Part 1: Solving triangles, Applying Sine & Cosine Laws

1. In Figure 2, ΔREA represents position of the arm after 3.93 seconds into this phase of the upright spin. The angle formed by lower arm and the upper arm, $\angle REA$, has a measure of 150°. AE and ER have fixed length and are congruent to AS and SM respectively.

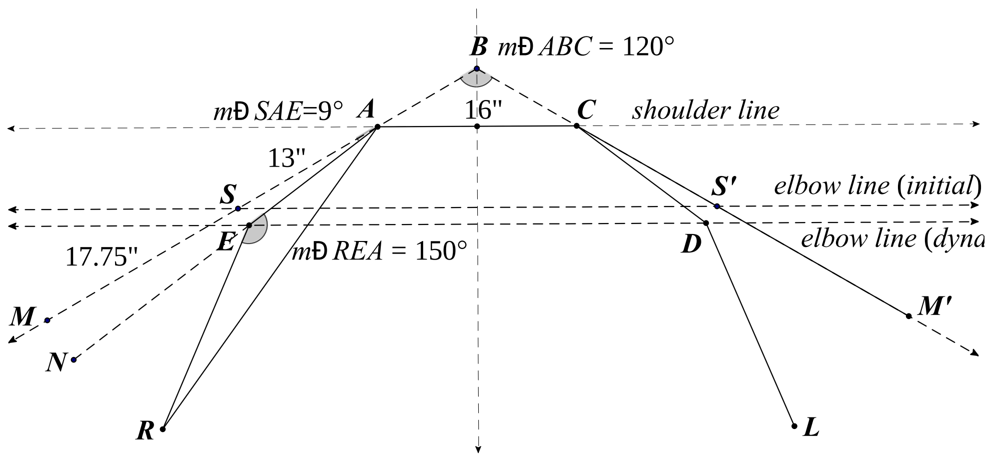


Figure 2. Upright spin model at 3.93 seconds.

a) Solve ΔREA completely: determine all of the missing angle measurements and segment lengths.
b) Find the area of ΔREA.
c) What is the difference between AM and AR?

2. During another part of the spin, $\angle REA = 100°$. (this occurs after 11.07 seconds; see Figure 3).

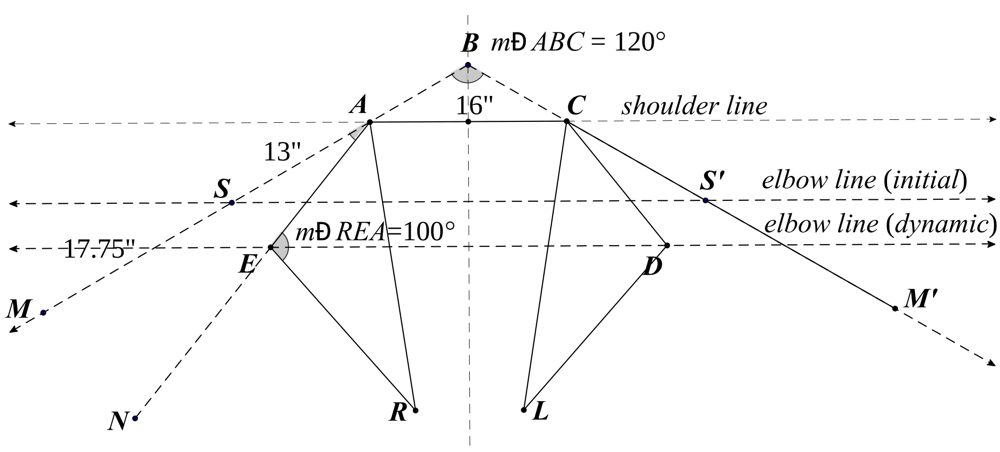


Figure 3. Upright spin model at 11.07 seconds.

a) Solve Δ*REA* completely: determine all of the missing angle measurements and segment lengths.
b) Find the area of Δ*REA*.
c) What is the difference between *AM* and *AR*?
d) Compare your answers to 1b and 2b. Does the area change? Explain why or why not.
e) Compare your answers to 1c and 2c. Does the distance *AR* change? Explain why or why not.

Part 2: Circular geometry, length of an arc

3. Due to a fact that the skater's upper arm lengths are fixed, one can conclude that points *S* and *E* are points on the same circle with its center at point *A*. Likewise, since the skater's lower arm lengths are fixed, *M* and *N* also are points on the same circle with its center at point *E* (see Figure 4).

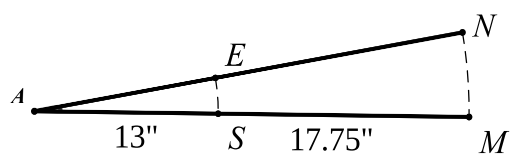


Figure 4. Isolated diagram depicting points A, S, E, M, and N.

a) Find the length of circular arcs *SE* and *MN* at both times: 3.93 seconds and 11.07 sec.
b) Find the difference between two ratios of *SE/MN* at both times. What does your answer mean?

Part 3: Transformations, interior angles of polygons

4. During the upright spin, skater's hands cross each other. In Figure 5, we depict when the left and right hands first touch each other. Note that the points R and L coincide, and we refer to them in this problem as point R.

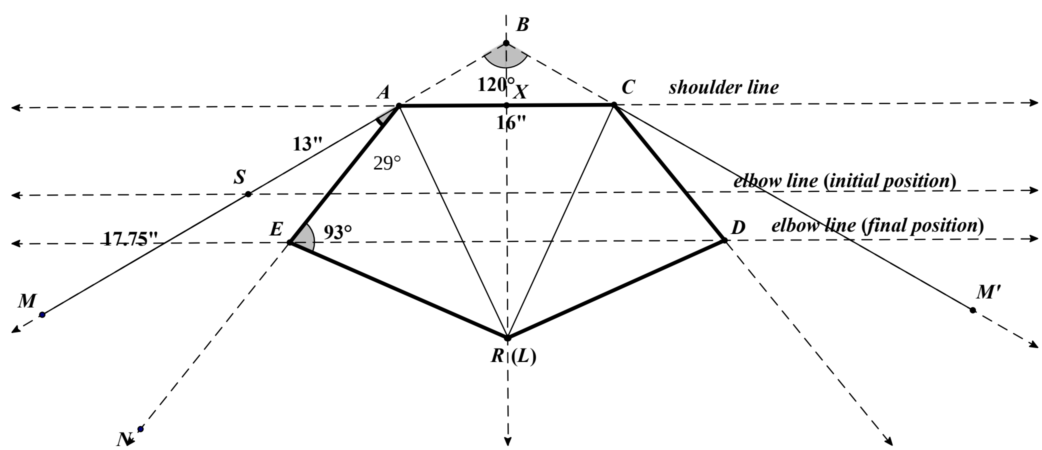


Figure 5. Upright spin arm model when the hands initially touch.

a) Does pentagon ACDRE have symmetry? If so, what type? Explain.
b) Find the measures of all the interior angles of the pentagon.
c) Find the length of diagonal *AR* of the pentagon.
d) Calculate the area of pentagon ACDRE.

5. [BONUS] Find the displacement of the elbow line, from its initial position (See Table 1, first row when time is 0 seconds) to the position when both arms are crossed and collinear with the elbows (See Figure 6).

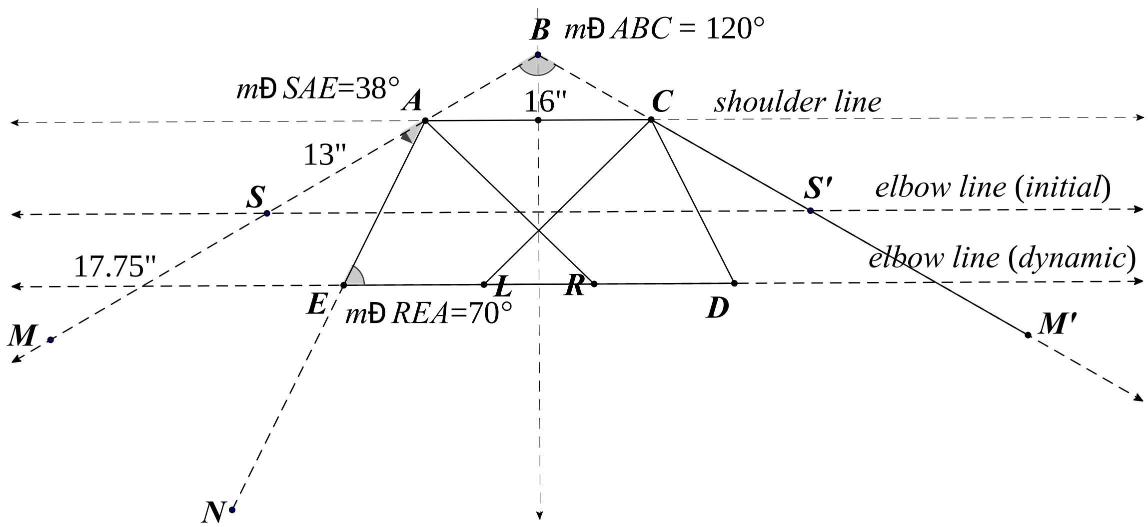


Figure 6. Upright spin model when arms are collinear with elbows.

Solutions to "Geometry & trigonometry in an upright spin"

Question 1: From ΔERA: ER = 17.75" and EA = 13", $\angle REA = 150°$;

a) By the Cosine Law, $RA^2 = RE^2 + EA^2 - 2 \cdot RE \cdot EA \cdot \cos\angle REA$
   By substituting the given measurements, $\boldsymbol{RA \approx 29.7277}$"
   Then, using the Sine Law, $\frac{RA}{\sin\angle REA} = \frac{EA}{\sin\angle ERA}$ and $\sin\angle ERA = \frac{EA\cdot\sin\angle REA}{RA}$. By substituting the given measurements and found measure of RA, $\boldsymbol{m\angle ERA \approx 12.6298°}$. Applying the Triangle Angle Sum Theorem, $m\angle EAR + m\angle REA + m\angle ERA = 180°$ and $m\angle EAR = 180° - (m\angle REA + m\angle ERA)$. By substituting known measurements, $\boldsymbol{m\angle EAR = 17.3702°}$.

b) There are multiple ways of determining the area of a triangle. In the solutions below, we have found the area of ΔREA using three different methods, based on the information that is selected. Note: All of the solutions agree to the nearest hundredth. The discrepancies are due to internal rounding errors.

First solution, SAS, using Area $\Delta = \frac{1}{2}ab\sin\angle C$

$$Area_{\Delta REA} = \frac{1}{2}RE \cdot EA \cdot \sin\angle REA \boxed{\approx 57.6875\ sq.in}$$

Second solution, SSS, using Heron's formula Area $\Delta = \sqrt{p(p-a)(p-b)(p-c)}$, where $p = \frac{a+b+c}{2}$

So, $p = \frac{RE+EA+RA}{2} \approx 30.2389"$

Then, $\boxed{Area_{\Delta REA} \approx 57.6893 sq.in.}$

Third solution, ASA or AAS, using one side and three angles $Area_{\Delta ABC} = \frac{a^2\sin\angle B\sin\angle C}{2\sin\angle A}$

$Area_{\Delta REA} = \frac{RA^2\sin\angle ERA\sin\angle EAR}{2\sin\angle REA} \boxed{\approx 57.6876 sq.in.}$

c) The difference $AM - AR = (AE + EM) - AR$; $\boldsymbol{AM - AR \approx 1.0223}$."

Question 2: From ΔERA: ER = 17.75" and EA = 13", $\angle REA = 100°$;

a) according to the Cosine Law, $RA^2 = RE^2 + EA^2 - 2RE \cdot EA \cdot Cos\angle REA$
   Substituting the constant values provided,

$$RA^2 = 17.75^2 + 13^2 - 2 \cdot 17.75 \cdot 13 \cdot Cos\angle 100°$$
$$RA \approx 23.7529"$$

   according to the Sine Law, $\frac{RA}{Sin\ \angle REA} = \frac{EA}{Sin\angle ERA}$ and $\frac{23.7529"}{Sin\ 100°} = \frac{13"}{Sin\angle ERA}$, so $m\angle ERA \approx 32.6147°$.
   They, using the angle sum of a triangle,

$$m\angle ERA \approx 180° - 100° - 32.6147° \approx 47.3853°$$

b) Area $\Delta ERA = \frac{1}{2}RE \cdot EA \cdot Sin\angle REA = \frac{1}{2}17.75\cdot 13 \cdot Sin100° \approx 113.6222\ \text{in}^2$

c) $AM - AR = (13{+}17.72) - 23.8 = 6.9971"$

d) The area of ΔERA is increasing since an angle formed by the static length sides is also increasing

e) With time passing, the distance AR is decreasing.

Question 3

a) The following angle values were recorded from the animation at two particular instances:
in 3.93 seconds $\angle SAE = 9°$, and in 11.07 seconds $\angle SAE = 26°$.

To find the length of a circular arc we used the formula: $\overset{\frown}{XY} = 2\pi R\frac{central\ angle}{360°}$.
Using the values recorded in 3.93 seconds, measures of the two arcs are:
$\overset{\frown}{SE} = 2\pi(13^")\frac{9°}{360°} \approx \mathbf{2.0420}"$ and $\overset{\frown}{MN} = 2\pi(13^" + 17.75")\frac{9°}{360°} \approx \mathbf{4.8302}"$.
Using the values recorded in 11.07 seconds, measures of two arcs are:
$\overset{\frown}{SE} = 2\pi(13^")\frac{26°}{360°} \approx \mathbf{5.8992}"$ and $\overset{\frown}{MN} = 2\pi(13^" + 17.75")\frac{26°}{360°} \approx \mathbf{13.9539}"$.
b)At 3.93 seconds, the ratio $\frac{\overset{\frown}{SE}}{\overset{\frown}{MN}} = \frac{2.0420"}{4.8302"} \approx 0.4228$; and at 11.07 seconds the ratio $\frac{\overset{\frown}{SE}}{\overset{\frown}{MN}} = \frac{5.8992"}{13.9539"} \approx$ 0.4228. Thus, the ratio of the corresponding arc lengths does not change over time. It means that in this model, the displacement of the elbow changes at the same rate that the displacement of the hand does.

Question 4

a) Yes, there is only one line of symmetry. A shape formed by two hands, bended at shoulder and elbow points, is a pentagon *ABCDRE*. This pentagon has two pairs of congruent sides: $AE \cong CD, ER \cong DR$. This line is a perpendicular bisector of shoulder line segment, labeled as *AC* in the diagram.

b) The hands touch each other at **12.7 seconds**. At this instant in time, $m\angle \boldsymbol{SAE} = \mathbf{29}°$ and $\boldsymbol{m\angle REA} = \mathbf{93}°$. By solving isosceles ΔABC, we find that $\angle BAC = 30°$. $\angle EAC$ is supplementary to $\angle BAC$ and $\angle SAE$. Thus, using the property of supplementary angles, $m\angle EAC = 180° - (m\angle BAC + m\angle SAE)$. By substituting the known values, $m\angle \boldsymbol{EAC} = \mathbf{121°.}$
Pentagon *ACDRE* can be divided into three triangles: $\Delta EAR, \Delta ACR,$ and ΔCDR.
As stated above, *BL* is a perpendicular bisector to *AC*. Then, from ΔACR, is easy to show that $AR \cong CR$ since points *A* and *C* are equidistant to point *R*. In addition, $\angle RAC \cong \angle RCA$.
According to the SSS principle $\Delta EAR \cong \Delta CDR$, and by the CPCTC rule corresponding internal angles are also congruent: $\angle EAR \cong \angle RCD$ and $\angle REA \cong \angle CDR$.
Thus, $m\angle \boldsymbol{CDR} = \mathbf{93}°$.
Since $m\angle EAC = m\angle EAR + m\angle RAC$ and $\text{m}\angle ACD = m\angle ACR + m\angle RCD$, because each consists of congruent parts $\angle EAC \cong \angle ACD$. Thus, $\angle \boldsymbol{ACD} = \mathbf{121}°$.
Finally, to find $\text{m}\angle DRE$, we subtract from a sum of all internal angles of *ACDRE* four angles identified above. So, the sum of all internal angles of *ACDRE* equals to $3 \cdot (180°) = 540°$, since pentagon consists of three triangles. And, $\text{m}\angle DRE = 540° - m\angle REA - m\angle CDR - m\angle EAR - m\angle ACD = 540° - 2(93°) - 2(121°)$. Thus, $\text{m}\angle \boldsymbol{DRE} = \mathbf{112}°$.

c) From ΔEAR, according with the Cosine Law:
$$AR^2 = AE^2 + ER^2 - 2AE \cdot ER \cdot \cos\angle REA$$
By substituting the known values, $\boldsymbol{AR} \approx \mathbf{22.5436"}.$

d) To find the area of pentagon *ACDRE*, we will find areas of three triangles: $\Delta EAR, \Delta ACR,$ and ΔCDR. $Area_{\Delta EAR} = \frac{1}{2}AE \cdot ER \cdot \sin\angle REA$, and by substituting known values, we find $Area_{\Delta EAR} \approx 115.2169°$. $Area_{\Delta CDR} \approx 115.2169°$, since $\Delta EAR \cong \Delta CDR$ as shown previously. Using Heron's formula $Area_{\Delta ACR} = \sqrt{p(p-AC)(p-CR)(p-RA)}$, where $p = \frac{AC+CR+RA}{2}$: by substituting the known values, $p \approx 26.6468"$, and $Area_{\Delta ACR} \approx 115.2169$ in^2.
Therefore, $\boldsymbol{Area_{ACDRE}} \approx \mathbf{345.6507}$ in^2.

Question 5

The displacement of the elbow line can be represented by the distance between lines *SS*' and *ED*.

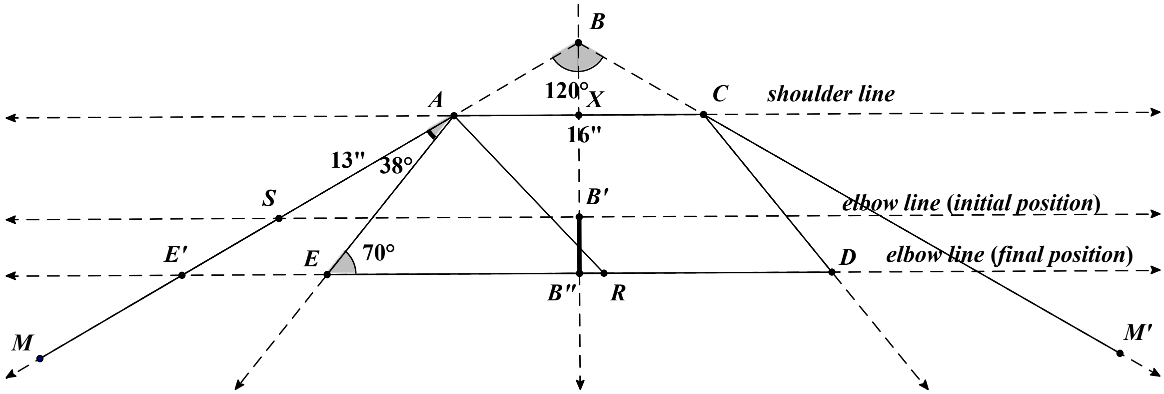


Figure 7. Model corresponding with solution to Question 5.

- Let *B*' represent an intersection of an initial elbow line *SS*' and an angle bisector of $\angle SBS'$. Let *B*" represent an intersection of a final elbow line *ED* and the angle bisector of $\angle SBS'$.
  $\Delta E'BD'$ is an isosceles, since $BE' \cong BD'$ [$AB \cong BC$, previously proven that ΔABC is also isosceles; and, $AE' \cong CD'$ representing distance from shoulder to wrist].
- $\Delta ABC \sim \Delta SBS' \sim \Delta E'BD'$ by the SAS theorem: $\angle B$ is common, and *AB*, *SB*, *E'B* are proportional as well as *BC*, *BS*' and *BD*'. Thus, according to Thales Theorem $AC \parallel SS' \parallel E'D'$.
- From ΔABC, it is given that $\angle B = 120°$, and according to the property of angles of the isosceles triangle, an internal angle at the base is (180°-120°)/2= 30°. Applying the Sine Law, $\frac{AB}{\sin\angle C} = \frac{AC}{\sin\angle B}$ and $\frac{AB}{\sin 30°} = \frac{16"}{\sin 120°}$,
  we find AB ≈9.2376". And by solving a special $90° - 30° - 60°$ ΔABX, $BX \approx 4.6188"$. Then, $\Delta ABX \sim \Delta SBB'$, corresponding sides are proportional $\frac{BB'}{BX} = \frac{SB}{AB}$, and by substituting the known values, $\boldsymbol{BB'}$ **≈11.1188"**.
- From $\Delta E'AE$, $\angle AER = 70°$, $\angle E'AE = 38°(diagram)$, and $\angle E'EA = 110°(as\ supplementary\ to\ \angle AER)$
  Then using the Sine Law, $\frac{E'A}{\sin 110°} = \frac{AE}{\sin 32°}$, and $\boldsymbol{E'A}$ **≈23.0526"**.
- $\Delta E'BB" \sim \Delta SBSB'$, thus $\frac{B"B}{B'B} = \frac{E'B}{SB}$, solving for $\boldsymbol{B"B}$ **≈16.1451"**.
- Thus, *B"B'=B"B – BB',* and $\boldsymbol{B"B'}$ **≈5.0263"**.

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The Use of Words in the Mathematics Classroom

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Abstract

Effective communication is at the heart of quality mathematics teaching and learning, and the new South Carolina College- and Career-Ready Standards for Mathematics renew the call to focus on communication in the classroom. To learn mathematical words, students will look to the language used by their teachers. Helping students communicate mathematically requires teachers to be precise in their own explanations and to help students connect common language to vocabulary through exploring the roots of words.

Effective communication is at the heart of quality mathematics teaching and learning, and the new South Carolina College- and Career-Ready Standards for Mathematics renew the call to focus on how communication is used in the classroom. The Standards maintain that mathematically literate students should be able to "reflect on and provide thoughtful responses to the reasoning of others" (p. 20) as well as "use appropriate and precise mathematical language" (p. 20) which includes the use of vocabulary. To learn how to use mathematical words appropriately and precisely, students will look to the language used by their teachers. Specifically, the words teachers choose can influence what and how students learn, and, in turn, the words students use to communicate about mathematics can reflect what they know. Arguably, students do not fully understand a mathematical concept if they cannot communicate about it with precision and accuracy. Ball and Sleep (2007) maintain that "mathematical language is both mathematical content to be learned and [a] medium for learning mathematical content" (p. 13). To be mathematically literate, therefore, students must be well-versed in not just the use of numbers and algorithms but in the names and words used to describe the procedures and concepts of mathematics.

Discussion

Learning to use mathematical language correctly, however, can be a daunting task for students, particularly in the area of mastering vocabulary, because mathematical words can be both familiar and mysterious to the student. Many students come to mathematics knowing common words such as *limit* and *place* but must learn how to set aside their understanding to learn the specialized definitions these words have in mathematics (Hardy, 2008; Schleppegrell, 2007). In addition to learning new definitions for familiar words, students are also asked to learn words that are foreign to them such as *asymptote* and *polynomial*. Because of these demands, learning mathematical vocabulary can be a struggle for some students.

Another area of potential confusion for students in mastering mathematical vocabulary is the use of informal language in the mathematics classroom. In an effort to make mathematics relatable to students, teachers often use informal language to discuss and describe concepts. However, the language used can lack precision to fully describe the concept. Such imprecision can lead to faulty conclusions. For example, it is not uncommon to hear teachers discuss "*canceling out*" factors when reducing rational functions. The meaning that is intended is that the common factors as a fraction reduce to one; however, using the phrase "*canceling out*" can lead students to believe the result should be zero. This interpretation is understandable because the phrase does mean zero when used in other circumstances. For example, when someone cancels a debt, it means that the balance owed is zero. Another example of improper use of informal language in the mathematics classroom is the use of the phrase "*no slope*" when analyzing the slope of the line. It is not clear from the use of this phrase if the line has a slope of zero or if the slope is undefined which is critical to determining whether or not the line is horizontal or vertical. Therefore, teacher use of these ambiguous phrases can be confusing for students and can lead them to faulty conclusions.

Connecting the Roots of Words to Math and Everyday Language

Although mastering the precise use of mathematical vocabulary can be difficult for students, Rubenstein and Schwartz (2000) maintain that carefully examining the origin of the words can help students understand the concepts. For example, the word *perimeter* is composed of the roots "*per"* which means "around" and "*meter"* which means "measure"; therefore, the precise definition of *perimeter* is the measure around a figure (p. 665). Another example is the word *tangent* which comes from the root *"tangens"* which means touching; therefore, a line tangent to a circle touches the circle in one point (Rubenstein & Schwartz, 2000, p. 665).

To help students master vocabulary words like *perimeter* and *tangent*, teachers can also help students connect these specialized words to words used in everyday language. For example, the word *peripheral* means around an object; therefore, peripheral vision concerns what occurs around the center of focus. To connect the word *tangent* to common language, mathematics teachers can discuss with their students what it means for an object to be *tangible* or touchable (Rubenstein & Schwartz, 2000). By exploring the roots of mathematical words with students and helping students connect these words to their common language, teachers are making the mathematical content relatable to students in order to facilitate student understanding of the concepts. Essentially, exploring the connections between the roots of words and their mathematical and common usages can help students delve deeper into the mathematics.

Conclusion

At the heart of teaching mathematics effectively is the notion that we want to help our students fully understand the concepts. We want them to know more than the algorithms. We want them to be able to explain with precision the how and the why behind the steps they take. In the terms of the new South Carolina Standards for Mathematics, we want them to be mathematically literate. To be mathematically literate, they need to know the vocabulary of mathematics, but learning this vocabulary can be a difficult task. Helping our students navigate the obstacles of mastering vocabulary requires that teachers attend to precision in their own explanations as well as help students connect common language to vocabulary through exploring the roots of words. In so doing, teachers will be heeding the call to create students who understand mathematics deeply – who are mathematically literate.

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The Undisciplined Mind: An Honors Course in Mathematics and Humanities

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Abstract

This article describes a collaboratively taught honors course focused on the connections between mathematics and the humanities, developed in response to students' fear of the unknown. The authors wanted mathematics students to appreciate the humanities, and humanities students to appreciate rather than fear mathematics. While the class developed in ways the authors could not have anticipated, students in the course generally showed a deeper understanding and appreciation of a broad liberal arts education.

Teachers and scholars regularly comment on the value of interdisciplinary curricula and innovative interdisciplinary courses (e.g., McIntire, 2015; Ottino & Morson, 2016). In many cases, educators adopt an interdisciplinary approach in order to deepen students' appreciation for or understanding of a particular subject, putting one discipline in the service of another. For instance, Bahls (2009) describes his use of poetry in a Mathematics course as an effective means of teaching Math. Such courses clearly have tremendous benefits for students. However, in this article a different model of interdisciplinary course is described—one in which the subject is disciplinarity itself—and an argument is made that such a course can help students to become more critical, active learners by involving them in the ongoing conversation about interdisciplinarity and the purpose of higher education.

The authors of this article of the most frustrating and perennial complaints that college students voice about their general education classes is that they are useless. Humanities students, for instance, often complain that math is difficult, useless, and terrifying. Students in professional programs balk at the number of general education requirements they must "get out of the way" in order to begin their career training. Students in STEM fields too often cannot see the value in their humanities classes. In particular, many mathematics students often cannot write well and are determined <u>not</u> to write well – believing the ability to communicate in writing superfluous to a mathematics degree, despite the need for students to be able to write proofs, analyze data in context, and communicate that analysis in writing to others.

Students' apathy toward general education stands alongside the national discourse about higher education. Professionals in higher education hear constantly from politicians and pundits about the need for more STEM education to prepare students for the demands of the twenty-first century economy (e.g., Obama, 2011). Of course, some of these endorsements can themselves reinforce stereotypes of liberal arts classes as useless. In response, many have leapt to defend the liberal arts, both on pragmatic (Jay & Graff, 2012; Strauss, 2013; Brooks, 2016) and humane grounds (Vanhoutte, 2014; Weber, 2015). One effect of this back-and-forth is to undermine the visibility and power of interdisciplinary approaches to education. Further pressure arises from Southern Association of Colleges and Schools Commission on Colleges and other accrediting bodies who set required hours and require courses segregated in various disciplines. This challenges universities that wish to provide interesting, valuable content with college appropriate content possessing breadth and depth and still at a level accessible to all while reaching for interdisciplinarity.

However, from the perspective of the authors as professors at a small liberal arts university, rather than continuing the trend of mutual exclusivity between STEM and the humanities, this conversation should move more toward interdisciplinary cooperation and away from the strict segregation of the disciplines. It is also important for students to be part of this conversation, so the authors designed and taught a course within the Lander University Honors College, called "The Undisciplined Mind: A Polymathic Approach to Life, the Universe, and Everything." The course explored the specific relationship between Mathematics and the humanities, as well as the more general relationships among these fields and higher education (math was chosen as representative of STEM fields simply because this is Dr. Ryan's area of expertise). While these relationships provided the content for the course, the authors wanted students not simply to learn about the connections between math and the humanities, but more importantly for that learning to help them better appreciate the importance of academic work across disciplines. The authors believed that such appreciation should help students become more confident and active learners, willing to take risks in problem solving.

Initial Plans for the Course

The initial planning for the course was relatively uncomplicated thanks to supportive department chairs and deans who were enthusiastic at the thought of an interdisciplinary course for Honors students. The course was team taught, technically listed as two separate courses meeting at the same time in the same room: one satisfying the general education requirement for Honors freshmen and sophomores (HONS 211) and the other an Honors elective for juniors and seniors (HONS 390). Officially Dr. Ryan was the instructor of record for HONS 390 and Dr. Barnette for HONS 211, but both groups received exactly the same assignments, and both instructors responded to all student work. The professors met and determined grades by consensus.

The articles and books chosen for the course concerned the work of polymaths who were exceptionally skilled mathematicians. The variety of thinkers, cultures, and eras within the scope of the class was vast, and given the time constraints of a single semester discernment was needed in choosing texts for the class. This requirement meant narrowing the focus—rather than including material from Chinese, African, and Central and South American, as well as European approaches to mathematics and philosophical thought. Instead, the course focused on discussions of Saint Augustine of Hippo, Rene Descartes, Omar Khayyam; students read Augustine's *On Free Choice of the Will*, trans. Thomas Williams, Amir Aczel's *Descartes' Secret Notebook,* Fitzgerald's *Rubaiyat of Omar Khayyam,* and a portion of Descartes' *Discourse on the Method*. Discussions of the required texts were augmented with a series of shorter articles on the value of a Liberal Arts today.

Augustine was selected because he was likely unfamiliar to most of the students, he was not primarily a mathematician, but he expressed some fairly profound mathematical ideas in elegant and subtle ways. Descartes was chosen because his mathematical work includes many concepts which students have seen in their high school and college math classes, and because his work in philosophy had a major impact on Western thought. Amir Aczel's book looked as if it would be accessible and interesting and provide for discussion on the impact of culture and religious elements, specifically the Inquisition and a secret society called the Rosicrucians. Khayyam was chosen from the vast, eminent group of Middle Eastern mathematicians because he was not only an accomplished mathematician, but also a prolific and widely recognized poet.

As with any course, activities and reading assignments were designed to help students master the course material, but also to help them develop the problem solving skills and academic confidence that were part of the original conception of the need for this course. The authors felt that, more than anything, the course needed to provide students with a safe place to make mistakes (Gojak, 2013; Pennant, 2013). Because the students were members of the Honors College, they were accustomed to doing well in school (and to measuring their achievement mostly by their grades). Because of this it was important to provide them with activities they would not be immediately successful with in order to allow them to examine their own learning process.

Math Activities

Since the main goals for the course included developing courage, creativity, and problem solving skills in students and showing them that Mathematics, when divorced from their fears of failing grades, is no different from any other unfamiliar topic, Dr. Ryan designed several in-class mathematics activities. These involved mostly upper level mathematics topics which none of the students would have seen before. These activities were intended to present humanities students, whose exposure to mathematical thought may have been limited, with elegant examples of advanced mathematics. Reading assignments throughout the course made connections between such mathematical thought and the humanities.
On the first day of class Dr. Ryan gave a brief overview of mathematical logic necessary for formal proof. Then she reviewed the Euclidean postulates from high school geometry, explained the genesis of finite geometries and showed them how to construct the three point geometry from its axioms. The students were then asked to construct the four point, four line, and Fano's geometries from the given axioms on their own--discussing with their table partners. Dr. Ryan's plan included providing the students with colored pencils in order to increase their perception of the assignment as a game, rather than a "terrifying new mathematics problem." The students received the pencils with enthusiasm.

At first students expressed serious apprehension in the face of unfamiliar mathematics. However, Dr. Ryan asked them to remember they would not be graded on success and to persist with courage. The students were encouraged to think of the assignment as an art project and a logic exercise. They did very well with the four point and four line geometries. As demonstrated by the axioms and graph shown in Figure 1, Fano's geometry was considerably more challenging, and few students could complete many of the steps correctly. However, they remained positive and were proud of how far they had gotten when Dr. Ryan revealed the solution.

<u>Axioms for Fano's Geometry</u>

1) There exists at least one line.

2) Every line of the geometry has exactly three points on it.

3) Not all points of the geometry are on the same line.

4) For two distinct points, there exists exactly one line on both of them.

5) Each two lines have at least one point on both of them.

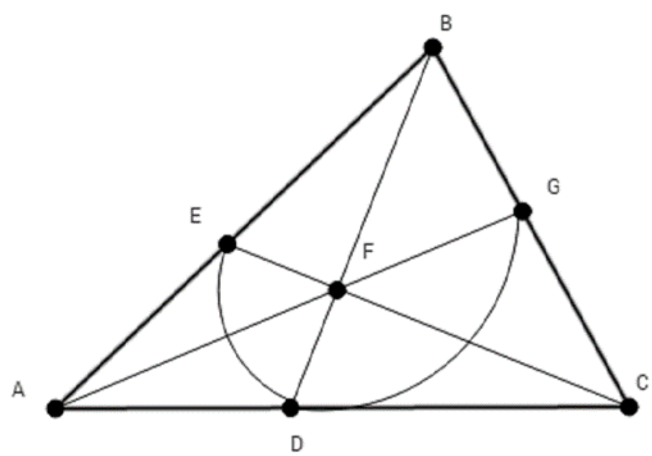

Figure 1

Another goal for the course was for students not to shrink from a challenge presented in an unfamiliar discipline. For the next math assignment the students were asked to read, outside of class, an excerpt from chapters 15 and 16 of Dahlke's (2008) *How to Succeed in College Mathematics* which explained how to read a mathematics textbook. In the following class, they were handed excerpts from Fraleigh's (1994) *Abstract Algebra* in which groups are defined. They were told that this was senior level, math major material and asked to decipher the material and solve several problems, including completing the construction of the Klein – 4 and the Z_4 group tables, see Figures 2 and 3.

Z_4 Group

+	0	1	2	3
0	0	1	2	3
1	1	2	3	0
2	2	3	0	1
3	3	0	1	2

Figure 2

Klein-4 Group

*	e	a	b	c
e	e	a	b	C
a	a	e	c	b
b	b	c	e	a
c	c	b	a	e

Figure 3

As the students interacted with each other and the assignment. Dr. Ryan roamed among the groups and encouraged them to ask pertinent questions. She would not answer "*How* do you do this?" Nor were the students allowed to complain that they could not do it. They could, however, ask if their interpretation was correct or on the right track. The students were encouraged to try something, anything, to be willing to be wrong. The students' initial reactions were insecurity and a good bit of indignation. However, with encouragement, with many reminders that they all knew how to read well and that we were simply asking them to decipher unfamiliar writing, most of the students worked hard and were able to decipher a good bit of the information. One or two students refused to even try. However, those who gave it their best effort and communicated with the students around them in order to decipher the material made progress and, in the process, became increasingly confident and enthusiastic. The students who decided before even trying that they could not and would not do the assignment and that it was intrinsically worthless to them, reaped what they sowed: nothing but frustration.

At the end of the class meeting time, students wrote answers to several questions about their reactions and what they perceived as the value of the assignment. The assessed how much success they anticipated having when they began reading for the math exercises and what results they expected from their efforts, how collaborating with other people affected their accomplishments within the assignment, what skills and previous experiences they drew on in their efforts to complete the assignments, and, finally, what value the series had beyond gaining of mathematical knowledge.

The students responded in a variety of ways, expressing their initial trepidation and their gains in confidence through working the problems:

- "I didn't think I would understand it that much. I assumed that this high of a math level and curriculum would be difficult to comprehend. After reading, the hardest part was not knowing what the symbols meant. When I collaborated with others, I was able to pick up the pieces that I didn't understand. It made it easier to have another brain working with mine."
- "…I [learned to form] coherent questions to ask when I don't understand a foreign topic"
- "These exercises can also be applied to any form of reading and interpreting"
- "Technically the smartest people in the university and we get handed something and we're like oh no and we end up doing fine"
- "This sounds kinda bad….being in the honors [program] and not understanding something made me feel bad. Talking to other people who were also confused made me feel better."
- "It's bad to be wrong except in this class."

Many students during the course said their math teachers do not encourage them to be wrong. This is a failing of institutionalized mathematics education (though it is by no means unique to math). Students who are not allowed to be wrong, who fear ridicule when giving a wrong answer stop trying (Nayar, 2010).

Not all students, of course, were successful. One student declared that he expected to be successful, but "after reading a few paragraphs, I realized that I was way in over my head." His response to the value of collaboration was "None. If I cannot understand for myself and reach the answer myself, then I did not learn." He stated that the exercises had no value beyond a study of mathematics and "I don't see that it helped me one bit, especially when all my thoughts are going toward [the midterm paper], not solving senior level math." This was one of the students who refused to try- the failure, we think, being that he could not transcend the need for material to be directly relevant to his studies, his grade, and his degree. This is the trend of education, especially in light of new laws that restrict how financial aid can be applied to courses not directly related to one's major. The authors feel this further underlines the real need of courses like theirs—and for such courses to begin much earlier than in college. This need is addressed later.

Overall, the mathematics activities appeared to contributed to the open communication and the community feeling of the group because students were united in their struggles with the strange and unfamiliar, depending on each other for ideas and resources. Some of the activities worked better than others, and we will have more to say about this in a later section.

Writing

Students wrote a considerable amount during the course. They wrote weekly response papers, which were particularly useful because they allowed students to explore ideas from the readings in more depth than they could during large group discussions, and because they allowed the authors to be aware of any problems students may have had with the material. Meeting regularly to discuss and grade students' response papers provided an excellent framework for ongoing course evaluation and planning.

Another low-stakes writing assignment involved writing poetry. To help students understand the relationship between creativity and structure inherent in many disciplines, including both math and English, students composed poems with very strict constraints. For a simple exercise, students were asked to write a limerick or a haiku, and later were given more complex constraints. For example, students used a finite state automaton to write poems in which the only possible letter patterns were those found in the names of students in the class (for several more examples of poetic forms with mathematical constraints, see Bahls 2009). While some students were initially hesitant to write poems, as they were to engage with the math activities described above, many of them composed verses that were both clever and creative. One mathematics major stated at the beginning that she was "not at all creative," but by the end of the poetry assignments, she said she had surprised herself with what she could do and that she could never again think of herself as lacking creativity.

The midterm paper assignment was more formal: A traditional research paper in which students were asked to comment on the ongoing cultural debate over the value of the liberal arts. To begin, students were given a series of mainstream articles that representative of the current national debate (e.g., Delbanco 2012; McMurtie 2015; Petrilli 2014; Rivard 2015; Young 2015). Students took these articles as a starting point and to make their own contributions to the debate, drawing on what they had learned from Augustine, Descartes, and the other class activities.

Other Activities

While students completed a significant amount of writing both in and out of class, most class meetings were devoted to discussions and other interactive activities. Naturally, many classes were devoted to discussing the assigned readings, and these discussions typically focused on identifying and helping students understand the mathematical ideas present in or underlying texts that were not explicitly mathematical, such as Augustine's *On Free Choice of the Will* and Descartes' *Discourse*.

The class considered Augustine's views on thinking, noted the passages which were very mathematical, and considered the soul of human beings versus other animate creatures and the ability to reason. The concepts of the value of free will, the virtue of the "good will," and justice as a means of obtaining a happy life and the "highest good" fostered many discussions. The class also discussed Augustine's concept of "number" and its immutability: "…it never has been and never will be the case that seven plus three does not equal ten" (p. 44) and further noted that Augustine had a rudimentary sense of Calculus: "Now consider the beauty of a material object at rest; its numbers remain in place. Consider the beauty of a material object in motion: its numbers vary through time" (p. 61).

As these discussions proceeded, students considered how apparently diverse disciplines might be interrelated even in the modern university, a question which gave rise to several debates about the purpose of education. On the whole, students' ideas about the purpose of education fell into two broad instrumental categories: Education should make one more skilled and therefore more employable, or education should make one well-rounded leading to more flexibility and usefulness.

Interestingly, students were not generally persuaded by arguments—such as Augustine's in Book One of *On the Free Choice of the Will*—that education might be something to pursue for its own sake. Towards the end of the semester when students were asked to consider what the point of a liberal arts education might be if jobs and money were not an issue, a very animated discussion resulted. (See figure 4) However, no one mentioned the inherent value of curiosity until the very end.

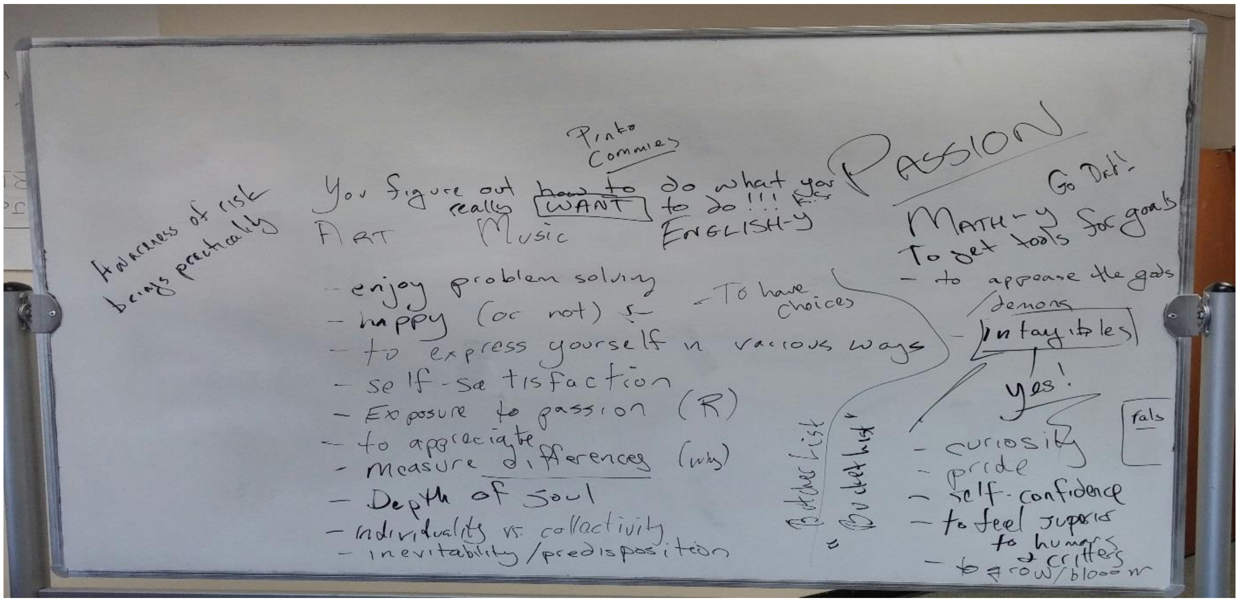

Figure 4: This photograph captures the notes taken during the class discussion of the non-economic value of the liberal arts education.

These discussions about the purpose of education led to the students' midterm project. As further preparation for that project, students also visited our university's archives. The archivist shared with them a variety of artifacts from the early years of the university (around the turn of the twentieth century), such as course catalogs, student handbooks, and letters written by the school's founder. Students examined these artifacts to determine how math and the humanities fit into Lander's curriculum in the past, and how the school's curriculum may have reflected or responded to larger cultural shifts in education over the last century.

In lieu of a research paper, students' end-of-term project was a research poster session. Students selected a person, society, or historical period whose work involved both math and the humanities, and researched their contributions to math, the humanities, and humanity in general. Students presented their research on posters because this medium gave them the opportunity to see and discuss all their peers' projects; this was important in helping students appreciate the breadth of topics relevant to the course. Student projects included broad overviews of how different cultures treated mathematics (including various Native American cultures, China, India, and the ancient Greeks), profiles of individuals (like John Nash, Richard Feynman, and Alan Turing), and broader "topics" (like origami, women in mathematics, and the link between math and psychology). Students presented their posters during Lander's Academic Showcase—a week-long conference of sorts, in which students from across the university present advanced projects in concurrent sessions. Two outside judges (both mathematicians from other universities) spoke with each student about their posters. Talking with the judges gave students the chance to discuss their work in depth and respond to challenging questions from evaluators who were not familiar with the work they had done all semester. This meant that students not only had to be experts in the topic of their research, but also had to be able to make a coherent argument about the relationship between math and humanities.

Obstacles

Although the course was quite successful overall, there were several obstacles during the semester, and several elements of the course will be changed the next time it is offered. Some students who did not see the relevance of the course material to their own discipline offered resistance, which manifested itself in lack of effort or in vocal complaints about particular assignments. While the authors recognized that this resistance was the precise attitude they hoped to address through the course and therefore expected it to some degree, they were surprised because the students who showed the most resistance were all from the group of advanced Honors students—that is, those for whom the course counted only as an Honors elective. This was the group for whom the course was the least useful in terms of meeting their graduation requirements, and therefore likely they enrolled in the course purely based on interest in the subject.

Another source of resistance came—more expectedly—from students new to the Honors College, whose response to the challenges we presented them was not overt resistance but rather a sort of smiling disengagement. Two students, in particular, remained enthusiastic in class discussions but seemed to put less and less effort into their written assignments as the course progressed. This may be attributed to the fact that these students had been used to high achievement in school and felt insecure about their abilities in a class with other advanced students. The authors met with each of these students repeatedly in order to help them with their work and increase their confidence, with mixed results. One of these students began to demonstrate both greater confidence and higher quality work during the final weeks of the semester; this improvement was especially notable on the student's final project.

Proposed Course Changes

As with any brand new course created half in advance and half on the fly, adjusting to the needs of the students and the exigencies of time constraints within a semester, there are specific changes that can be made to the course the next time it is offered in order to help reach all students more effectively. Primarily, structure needs to be tightened up a bit. Toward the end of the semester there were several unstructured days which originally were to be filled with "other mathematicians or cultures." In practice this turned out to be a very wobbly set of lectures. Furthermore, some of the students complained that this class was entirely "math and humanities" with no other disciplines represented. In all fairness, that was how the class was billed. However, to more fully address the value of a liberal arts education in today's world the authors plan to, next time, line up a series of guest speakers from other disciplines within the university such as sciences, business, and fine arts. Furthermore, the provost of the University will be invited to speak to our future class about "the big picture" – the way he sees the university in its entirety, all the disciplines twining together to make the Liberal Arts program. These short lectures will unite the fragmented bits of the course into a unified whole more successfully than two professors alone, rounding out students' understanding of the Liberal Arts.

The written response paper assignments limited those who preferred other forms of expression in ways they found overly restrictive. The next incarnation of this course, in addition to a substantial number of writing assignments, will have a few response paper assignments that allow for more creativity of form. A graphic novel response, a poem, story, or painting representing the student's thoughts on the course material might be appropriate in some of the units. Since the emphasis is on the variety and interdisciplinary nature of knowledge, giving the students a menu of responses seems valid and beneficial.

Initially these papers were due at the end of each week. However, because the course met a standard three day schedule, this meant the students were only discussing Mondays and Wednesdays and leaving Fridays out entirely. The quality of the papers at the end of the week showed they were writing them in the time before class. Moving dates to Mondays gave students a chance to include Friday and time to craft thoughtful responses.

There are many female mathematicians and polymaths who merit inclusion in the course. Next time we will include those who students found most interesting through their poster projects, such as Amalie (Emmy) Noether, Hypatia of Alexandria, and Elena Piscopia.

The end-of-year poster session involved sixteen posters and the two outside judges, as well as Drs. Barnette and Ryan in one 2-hour time period on a Friday afternoon. This turned out to be a tremendous underestimation of the time necessary for the judges to personally interact with the presenters, the material, and the posters. Even after almost four hours, both judges complained of being rushed and not getting complete pictures of the posters at the end. The students were exhausted, frustrated, and, in some cases needed to leave for other events. Next time, students will be divided into groups that can be evaluated in less than two hours. This will give the students a chance to view each other's work, keep the judges fresh, and preserve the rigor and interactive nature originally sought in the presentations.

Along with these structural changes, there need to be some changes to the reading assignments. Students responded well to the primary texts, especially Augustine's *On Free Choice of the Will* and Descartes' *Discourse*. However, Aczel's *Descartes' Secret Notebook* produced disappointingly thin discussions. The book was chosen because the professors felt it would meet the student's engagement level with biographical sources and, while the book presented Descartes' mathematical thought in a broad historical and social context, students complained, with some justification, that it moved too quickly and superficially over important mathematical issues and put too much emphasis on attempting to create historical intrigue. As a result, a more substantive, but still engaging biography will be chosen. The end of the semester reading schedule was left open with the expectation that the needs and interests of the class would guide selection of texts in the later weeks. However, as students began working on their individual poster projects, the course would have benefitted from a more explicit reading schedule.

The conversations occasioned by this course are worth having with students in university courses and at other levels. Students initially had trouble finding their way in such unfamiliar territory, but as the course progressed they interacted with enthusiasm and interest, with the material and each other. They left the course better prepared to take full advantage of their university education because they had a clearer sense of how the various components of that education are—and should be—connected, and how they might approach their education as critical, active learners. Such preparation is essential for students in the current climate of debate over the value of the liberal arts, mathematics, and higher education in general.

Conclusion

The structure of the course could be adapted to more traditional mathematics classes, adding papers and discussion to improve writing among math students (science students regularly write lab reports with analysis). Mathematics generally is not writing intensive beyond proofs until statistics and the analysis of data (Bahls, Mecklenburg-Faenger, Scott-Copses, & Warnick, 2011). Moreover, mathematics departments seldom offer courses which include thinking about mathematics in relation to its place in history- beyond its own development- but as it has influenced thought in every culture. This fails to present students with accurate and lively pictures of those who dabbled in math as a diversion from theology, history, poetry, and governance.

Furthermore, these conversations would be better begun earlier, even in elementary school. If students are asked to think broadly and make connections, seeing the humanities and mathematics and sciences as all interconnected, if they are asked to write in these disciplines and about these ideas from early ages, they will be better thinkers. They will be better writers. They will not treat subjects as concepts in isolation and thereby decide they are "just not good at math" or "just not very creative." They will see educational subjects and bars on a continuum or wedges on a pie chart – blending into one another and colors on a wheel.

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