

# THE MATHMATE

*The Official Journal of the  
South Carolina Council of Teachers of Mathematics*



**2019 SCCTM Fall Conference November 14 - 15, 2019**

**Greenville Convention Center, Greenville, South Carolina**

# THE MATHMATE

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## South Carolina Council of Teachers of Mathematics

2020 SCCTM Fall Conference - Greenville, SC

November 19, 2020 8:00 AM - November 20, 2020 4:00 PM  
Greenville Convention Center



*Seeing the Beauty and Wonder of Mathematics*

# THE MATHMATE

**Mission Statement:** The mission of The MathMate is to feature articles, about innovative mathematical classroom practices, important and timely educational issues, pedagogical methods, theoretical findings, significant mathematical ideas, and hands-on classroom activities and make this information accessible to students, educators and administrators.

**Submission Requirements:** All submissions are to be emailed to [scmathmate@gmail.com](mailto:scmathmate@gmail.com) as attachments along with a completed Submission Coversheet. The coversheet can be found at <http://scctm.org/The-MathMate>.

Submitted files must be saved as MSWord or PDF files. Pictures and diagrams must be saved as separate files and appropriately labeled. Authors are asked to not submit the same article to another publication while it is under review for The MathMate.

**Submission Deadlines:** The MathMate is published 3 times per year. Submissions received by November 1 will be considered for the January issue, March 1 for the May issue, and July 1 for the September issue.

**South Carolina Recertification Credit:** According to the SC Department of Education Renewal Credit matrix, <http://ed.sc.gov/educators/certification/certification-forms/forms/renewal-matrix> , the primary author of a peer reviewed journal article can earn 60 renewal credits.

**Subscriptions:** Active members of SCCTM receive online subscriptions to The MathMate as part of their memberships.

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# THE MATHMATE

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# MESSAGE FROM THE SCCTM PRESIDENT

Dear Members of SCCTM,

On behalf of the board, I would like to thank all those involved in the creation of our journal, The MathMate. I would especially like to thank Jennifer Thorsten and the various members who have supported the reviewing process and have aided authors seeking publication.

As this is the first edition of The MathMate in the new decade, I would like to share my excitement for all of the fantastic efforts being made by your SCCTM Executive Board. From the Dine & Discuss professional development which includes a virtual visit from Dr. Jo Boaler herself on February 20<sup>th</sup>, to the possible re-naming of our journal, and finishing with the Annual Conference in November, there are a great many endeavors being made on your, the members', behalf. We hope you will follow along and provide your suggestions and guidance throughout the year. We will be making some adjustments to our website to include the forum for the professional development on *Limitless Mind*, more information about your executive board team in case you need support or want to send feedback, and changing grant and scholarship submission processes to make them more accessible.

I want to take a moment to encourage you all to read Dr. Francis Su's new text *Mathematics for Human Flourishing* before our Annual Conference in November. His book will be of great support while you consider speaking at our conference this year. You will soon be receiving an invitation to submit proposals for the conference and we hope to see your submission! We can all learn so much from each other and provide the support we need to be amazing educators and flourish as professionals.

If you have any feedback or have questions, please feel free to reach out to me at [president@scctmconference.org](mailto:president@scctmconference.org).

Your friend in mathematics,

Ryan M. Higgins, Ph.D.

Coker University

Hartsville, SC

Darlington County

# ANNOUNCEMENTS

## Award Nomination Deadlines:

### **Outstanding Contributions to Mathematics Education Award**

Nomination deadline: July 15  
[scctm.org/Awards](http://scctm.org/Awards)

### **Richard W. Riley Award**

Nomination deadline: July 15  
[scctm.org/Awards](http://scctm.org/Awards)

## Scholarship Deadlines:

### **Preservice Scholarship**

Applications deadline: September 15  
[scctm.org/scholarships](http://scctm.org/scholarships)

### **Educator's Scholarship**

Application deadline: September 15  
[scctm.org/scholarships](http://scctm.org/scholarships)

## Membership News:

[Renew your NCTM membership online](#) and designate *South Carolina Council of Teachers of Mathematics* for the affiliate rebate.



If you would like your announcement to appear in the next issue of *The MathMate*, please email all information to [SCMathMate@gmail.com](mailto:SCMathMate@gmail.com). Announcements will be published at the discretion of *The MathMate* Editorial Board.

## Message from the Editor:

I would like to extend my sincere gratitude to all of the authors who submitted works for our publication and all of the reviewers of those submissions. Your contributions add to our collective knowledge and help to enhance the practice and art of mathematics teaching and learning in South Carolina and beyond.

There are wonderful things happening all over our great state, and I encourage you to share your stories! Did you present at the conference in November? Write a follow-up article for the *MathMate*. Have you and/or your colleagues made strides in implementing strategies for personalized learning, differentiation, collaborative problem-solving, mathematical modeling, etc.? Share your stories and extend the conversations so that all of us can benefit and grow!

In this issue I am proud to introduce a wonderful new column, *Manipulative Corner*, created by our President Ryan Higgins. In the inaugural edition she shares ideas that will make you want to wipe the dust off those Geoboards in the back of the supply closet! Please check out her article on page 13. If you have a favorite math manipulative, please share your insights and submit a contribution for this new, and hopefully reoccurring, column.

I hope that you enjoy this issue of the *MathMate*—my first issue as Editor, but it might be my last as well. SCCTM is considering changing the name of our publication as we combine this journal with the newsletter. If you have a suggestion for a name that embodies the creative spirit of sharing our mathematical stories, please submit it [HERE](#), or visit [www.scctm.org](http://www.scctm.org).

Please share your ideas for improvement and your submissions for inclusion with me at [scmathmate@gmail.com](mailto:scmathmate@gmail.com).

I look forward to our continued correspondence!

*Jen Thorsten*

# SCCTM EVENT

## Book Study and Dinner

**\$10 for current SCCTM Members**

**\$31 for new Members**

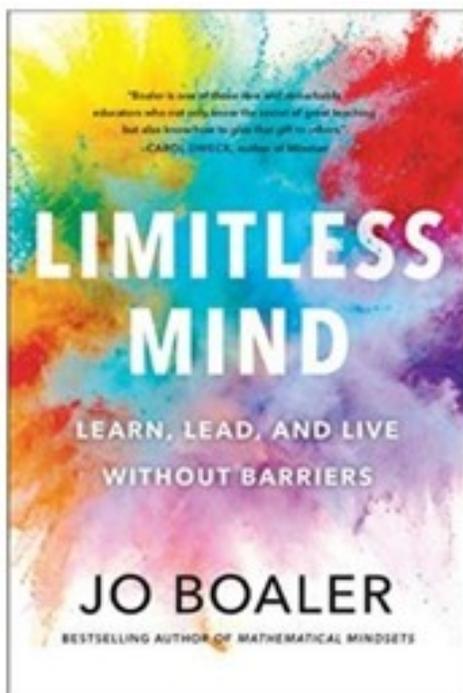
**Be sure to join us!**

**Bring Your Own Book**

*Limitless Mind: Learn, Lead, and Live Without Barriers*

Stanford University professor, bestselling author, and acclaimed educator Jo Boaler has spent decades studying the impact of beliefs and bias on education. In *Limitless Mind*, she explodes these myths and reveals the six keys to unlocking our boundless learning potential. Her research proves that those who achieve at the highest levels do not do so because of a genetic inclination toward any one skill but because of the keys that she reveals in the book. Our brains are not “fixed,” but entirely capable of change, growth, adaptability, and rewiring. Want to be fluent in mathematics? Learn a foreign language? Play the guitar? Write a book? The truth is not only that anyone at any age can learn anything, but the act of learning itself fundamentally changes who we are, and as Boaler argues so elegantly in the pages of this book, what we go on to achieve.

[Register here \(http://scctm.wildapricot.org/event-3619672\)](http://scctm.wildapricot.org/event-3619672) for the SCCTM Dine & Discuss Book Study February 20, 2020, on Jo Boaler’s book *Limitless Mind: Learn, Lead, and Live Without Barriers*.



# FINDING SUCCESS IN WORD PROBLEM SOLVING

by Raney Stogner

## Abstract

Students are consistently showing little to no confidence in solving word-problems which impacts their ability to complete them successfully. This is a study conducted on a group of third-grade students that showed to begin building success in word-problems, teachers must not only build number sense and problem-solving skills, but also provide meaningful and contextual problems. Number Talks may be the answer for our students!

## Background Information

The beginning of my study of word problems and Number Talks began in a third-grade classroom at a school about fourteen miles from the South Carolina State House. I was full-time student teaching and it was during the second week that my students were taking a math test on multiplication and division of the factor five. They were all diligently working, using the skills and strategies their teacher had taught them to use on their own. However, when they began nearing the end of the test, and the word problems revealed themselves, every hand in the room went up. Every child in the room needed help deciphering what the word problem was asking them to do. The questions, "I'm confused," "What am I supposed to do – multiply or divide?" "Can you help me?" quickly turned into, "Oh! Duh!" "I can do that!" as we walked around and helped clarify without giving away the answer. From that day forward, I began to wonder why students, including myself when I was in elementary school, are so afraid of word problems or why they think they are not capable of solving them. Problem solving represents the foundations of all mathematical activities and frequently these activities have been qualified as very difficult, complex, and distressing for many learners, and especially when they face math word problem solving (Eda Vula, Rrezarta Avdyli, Valbona Berisha, Blerim Saqipi, Shpetim Elezi, 2017, p.51). I wondered: How can students become more confident and successful in solving word problems? I began to gather more information

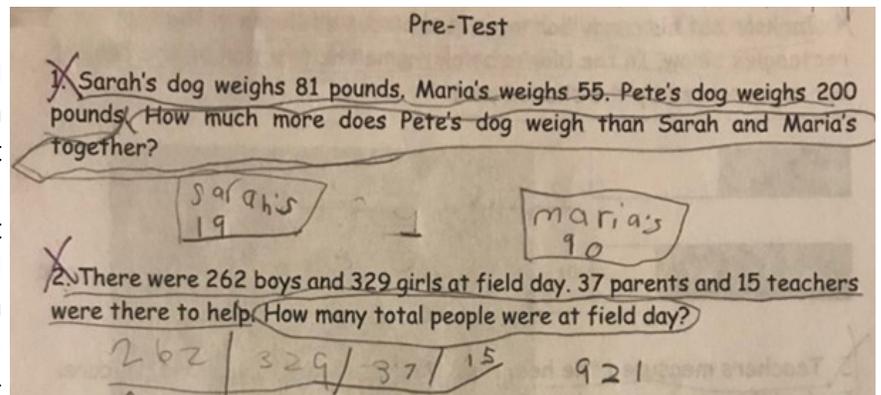


Figure 1

concerning their abilities and feelings towards word problems.

## Data Collection

I started with a pre-test that had about 7-word problems – one from each strand of the mathematics standards for third grade (Figure 1). The numbers or calculations were simple to ensure assessment of the ability to decipher word problems. After scoring the pre-tests, all 19 students scored in a range that placed them in the category "Not Proficient," meaning they earned fewer than 9 out of 12 points. Students who got almost all the problems wrong were having trouble figuring out what the word problems were asking them to do. These students know how to complete the computations once the operation to be completed is revealed to them.

On the pre-test, of the 8 students who scored a 0-4 out of 12 points, placing them in the "Not Likely" category, 5 of them are reading below grade level. The conclusion can be drawn that these students

could easily have not been able to do the math because they simply cannot comprehend the problem in written form. In, “The Impact of Metacognitive Strategies and Self-Regulating Processes of Solving Math Word Problems,” the authors state that reading comprehension is essential in solving math word problems (Eda Vula, Rrezarta Avdyli, Valbona Berisha, Blerim Saqipi, Shpetim Elezi, 2017, p.52). For these students that are reading below grade level, lower level reading is impacting their ability in other subjects like mathematics.

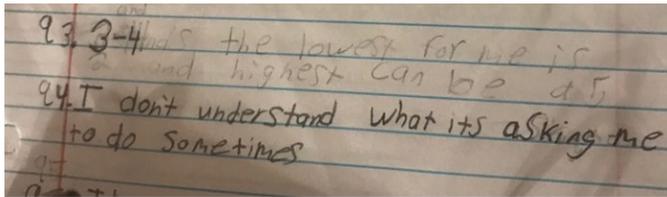


Figure 1.1

In addition to the pre-test, an exit slip was given that asked the question, “What do you think is the most difficult part of solving a word problem?” Of 19 students, 3 students said the most difficult part was “reading it” or “knowing the words.” The other 16 said the most difficult part was figuring out what to do to solve it (Figure 1.1) A strong support in comprehending the terminology used in word problems along with the unknown words is of great importance for learners’ success (Eda Vula, Rrezarta Avdyli, Valbona Berisha, Blerim Saqipi, Shpetim Elezi, 2017, p.54). Mathematical vocabulary is a key factor, which helps learners to improve their skills in solving math word problems.

### New Strategies Implemented

Through the pre-test and/or exit slips, multiple students expressed or showed the amount of confusion and anxiety they have in trying to decide what to do to solve the problem. I implemented the 5-finger rule:

- 1- Read the problem
- 2- Underline important information (mainly key words)
- 3- Circle the question

4- Decide which operation(s) are needed to complete

5- Solve

During math small groups, we worked on word-problems where students were required to complete the 5-finger rule each time. In the midst of instruction, I gave students ample opportunities to practice analyzing word problems – without solving them – to get comfortable and better at simply uncovering what is desired of them. After about a week and a half of implementing these strategies, I was able to scaffold the students who are stronger in word problems into being able to say aloud what is desired of them without requiring them to complete the 5-finger rule. They were able to identify the key words and depict single-step problems from multi-step problems. For the other students, they were encouraged to continue to use the 5-finger rule and they found it made solving the problem easier.

When planning instructionally, I created and gave students word problems they could associate or connect with personally to grow confidence in reading word problems. According to Scaffolding Language, Scaffolding Literacy, when you are unable to bring personal knowledge and understanding of a topic to a text, you are effectively robbed of the ability to make use of a key resource for reading: what you already know (Gibbons, 2002, Chapter 5). Through reading problems that contain topics with which this group of students is comfortable like basketball, music, etc., they could focus less on trying to read and articulate words but show what they know mathematically. In doing so, every day before math began, we completed a class word-problem. The students who showed success in the 5-finger rule and key words, were able to solve the problem without being required to underline, circle, etc. Students who had not quite mastered these strategies, were supported and guided as needed.

### Problems Finding Time for Word Problems

I noticed that other than the times specifically set aside for word problems, students were choosing

to not complete them during regular math time. Some students even purposely stopped working when they reached the word problems in their math books. Christopher Masullo once said in his article, "What's the Problem with Word Problems?" that "problem-solving is not only one of the most important components of the study of mathematics; it permeates all aspects of life, including the professional world." (2017) As a class, we decided to make a conscious effort to always complete the hardest things first – and not just in math. For instance, if they had the option to complete one thing before another, I encouraged them to complete the most difficult item first, so they could take advantage of the time they had with me there while I was able to help them when needed. In math, we began completing the word problems in the workbook first, before moving to the standard problems. When a few students discussed with the class that they needed to know how to solve word problems not only to do well on assignments/tests, but also to be able to problem-solve and critically think in life, the rest of the class agreed. At first, this was a struggle. Sometimes we were only able to complete one problem a day, and there were lots of frustrated students, but as the weeks moved forward, students automatically knew the routine: complete all the word problems, then move to the standard problems, and if they did not reach the standard problems, it was acceptable. Masullo

says that problem solving permeates all aspects of life (2017), and I expressed his thoughts to my students, and it enabled me to encourage them to persevere when they thought they were stuck and did not know what to do – even outside of mathematics.

### Activity to Gain Interest in Word Problems - Peer Problem Friday

Thus far, I had implemented strategies that could help students identify the steps they needed to take in order to get the right answer. However, I still felt students weren't connecting to the problems and it was impacting their ability to read them. During a week's span, I observed my students' engagement while completing the word problems I provided. On average, 12-15 students were engaged the entire time during whole-group solving. My lower students were still disengaged, and I knew I had to find a way to help them want to complete the problems. Moving forward, every Friday I used another activity to immerse them in problems that are contextually meaningful to them: students created word problems for each other (Figure 2). Instead of me creating the problems I thought they would connect with, I wanted them to create their own connection. We called this activity "Peer Problem Friday." The whole-class problems, along with this activity, enabled them to start on one of the lower Bloom's Taxonomy Levels, then gradually move

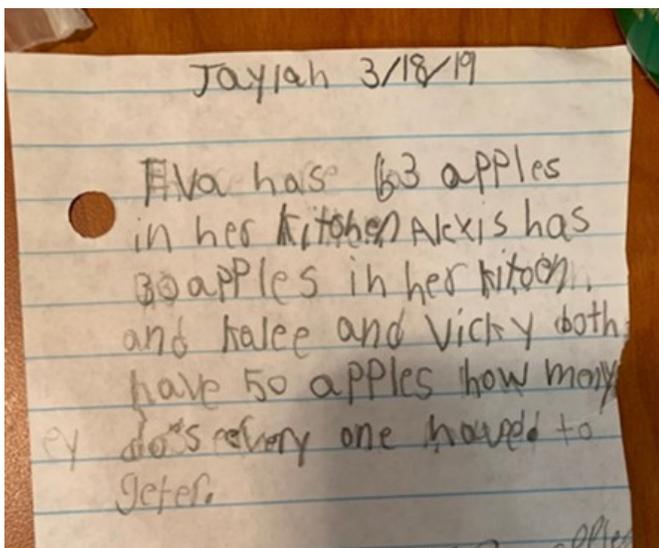


Figure 2

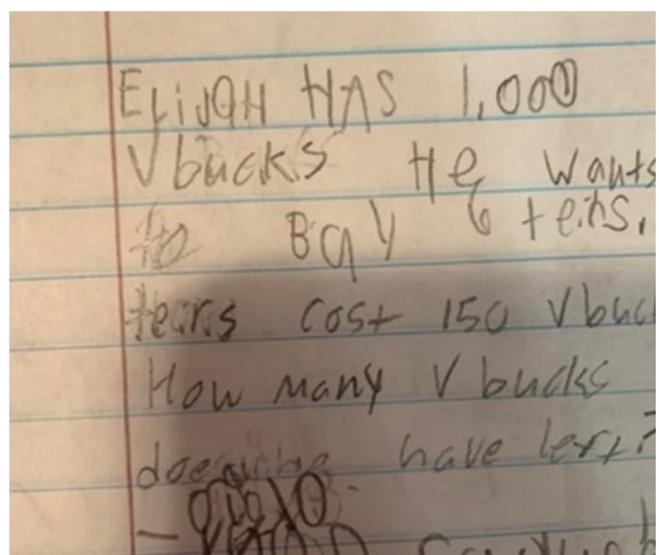


Figure 2.1

to a higher level by creating their own word problems in which they felt personal connections. Every student thoroughly enjoyed Peer Problem Friday – especially my struggling readers. Student “B” resented any math problem that had anything other than two numbers in which he was told the operation to complete for them. The first Friday we had Peer Problem Friday, I told him to write about anything he wanted – he chose Fortnite. Every Friday after that, he wrote about Fortnite, but created more difficult problems each time (Figure 2.1) He, and even his peers, commented on his ability to participate. “Woah! ‘B,’ your problem was hard today!” his peer said after a long fifteen minutes trying to solve.

### Numberless Number Talks

In the midst of all said strategies, I was also conducting daily Number Talks. Unexpectedly, I noticed the Number Talks were helping one of my lowest readers build confidence in math in general (not just word-problems) and another one of my most disengaged students was fully engaged the entire ten minutes of each Number Talk. Student “B” was one of these students. He was willing and excited to share the mental math he was completing. We completed Number Talks that contained just numbers for about three or four weeks. I noticed that the students were getting used to completing the mental math, and I was able to introduce many concepts that would normally be introduced in fourth or fifth grade, such as exponents.

Since noticing Number Talks were helping my students with number sense, being able to explain

their thinking, and much more, I also wondered if Number Talks could potentially improve their success in word-problem solving, too. I began Numberless Number Talks. This entails a word problem being placed on the board with no numbers. Discussions are had around the content of the word problem, what is needed to solve it, what changed after numbers are inserted, etc. According to, “Number Talks Build Numerical Reasoning,” keeping the discussion focused on the important mathematics and helping students learn to structure wonderings during a Number Talk is essential to ensuring that the conversation is meaningful (Parrish, 2011, p.204).

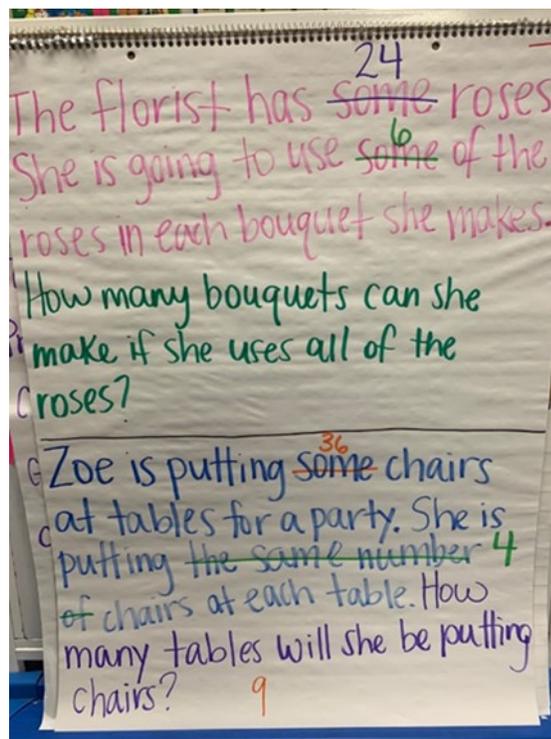


Figure 3

I strongly focused on this concept as we moved forward with Numberless Number Talks. I asked students what they were thinking about as they read the problem, what they were wondering, what they needed to know, etc. As students began wondering where the numbers were or how they were supposed to solve the problem without numbers, I began to coach them into expressing their wonderings meaningfully, so their classmates could benefit from them and begin to wonder as well. Conversations were had as words like “some” were crossed out and numbers were inserted, and as the question was placed at the end of the problem. Figure 3 shows two examples of Numberless Number Talks we completed as a class. Each color change is a time something was added to the problem. As a class, we wondered,

discussed, and solved the problem. I noticed my two lower students, “B” included, were immediately disengaged again. They went from being excited to participate in Number Talks, to not paying attention at all. “B” sometimes even caused disruptions. This provided implications for me that these actions were because they were not comfortable with what was now being covered during Number Talks.

In the short time I had left to address this and take action, I focused on giving these students ample support and positive feedback when they did show they were interested. I said things like, “Thank you for following directions and reading the problem, ‘B’.” The disruptions were eliminated as we completed a few together, but the participation was still slim to none. These actions led me to believe that over time Numberless Number Talks could be a time where these students again enjoyed and actively contributed – once they felt they were able.

### **Final Conclusions**

As I finished my semester with my 3rd graders, I was now wondering: Can Number Talks and Numberless Number Talks improve success in word-problem solving? As I enter my first classroom, I am going to begin by conducting daily Numberless Number Talks. I would encourage other teachers who have noticed their students struggling with word problems to do the same. It is our job to observe student participation and engagement and check for correlation when working with word-problems on a regular basis. It is important to continue to increase difficulty – creating Number Talks that require multiple steps to solve. Additionally, pairing lower students with higher students when completing partner discussions during Number Talks can increase thinking of all stu-

dents. I have observed the impact Number Talks can have on student thinking and learning, and I whole-heartedly believe these discussions, along with Numberless Number Talks, can improve our students’ success in solving word problems.

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### **About the Author**

Raney Stogner is earning a Master’s Degree in Education with a focus in Literacy from the University of South Carolina in Columbia, South Carolina. She conducted the action research described in this article during the senior year of her undergraduate degree.

# MANIPULATIVE CORNER

By Ryan Higgins

One of the priorities of SCCTM is to promote research-based, high quality mathematics teaching and learning in our state; in order to do so, we utilize resources such as NCTM's *Principles to Actions: Ensuring Mathematical Success for All* (NCTM, 2014). To summarize in a ridiculously short manner, the text describes best practices for mathematics teaching and learning as teaching mathematics in a way that involves thoughtful and demanding tasks, questions and inquiry techniques. One way teachers exhibit these great practices is through the use of equally great manipulatives. So...Welcome to the "Manipulative Corner"! Here, we will share ideas for manipulatives we love which inspire learning. In each edition, we will share a manipulative and several great ways to use it within a combination of multiple grade bands: Pre-K-2, 3-5, 6-8 and 9-12.

For our first Manipulative Corner, I'd like to ask, "Who doesn't love a Geoboard?!" No matter the student's age, a Geoboard is an exceptional tool and fun for everyone. I must admit, I've seen a great many of these gathering dust over the years. I know, I know...RUBBER-BANDS!! But after providing your students with very particular expectations for their use, Geoboards are great learning and teaching tools.

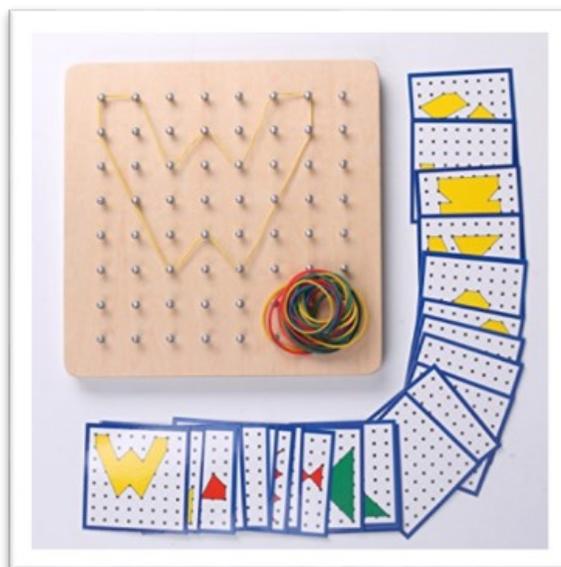
PreK to 2nd grade can use Geoboards to create and discover by doing some of the following activities:

- ◆ Explore attributes of shapes,
- ◆ Show symmetry,
- ◆ Construct numbers/letters, and
- ◆ Use an area model to develop early multiplication skills.
- ◆ Bonus: Students use fine motor pincer grasp in order to manipulate those rubber-bands. So, while I know the temptation may be to switch to a digital model of a Geoboard, consider the great benefits of the tangible board itself.

IDEA: A fantastic inquiry-based mathematics center for Pre-K and Kindergarten requires one Geoboard for each student allowed at the center at one time and cards with numbers on them. Place the numbers with coordinating dots for one-to-one correspondence at the center and have children work on recreating the number. Regardless of whether the child recognizes the number by memory, the child is working on the creation of the shape itself for a future reference point.



Pictures from [google.com](https://www.google.com), Google search: *geoboard*



3<sup>rd</sup> to 5<sup>th</sup> graders (and beyond) can work on multiple standards using this one manipulative. For example, the following list is short but includes a great deal of the standards in each of these grades.

- ◆ area models for shapes,
- ◆ introductory fraction skills (including iterating and partitioning),
- ◆ equal parts separation of shapes, and
- ◆ equivalent fractions and comparison of fractions.

IDEA: With children in pairs, present them with a Geoboard or its digital equivalent and fraction cards (i.e.  $\frac{1}{2}$  and  $\frac{3}{4}$ ). Ask each partner to construct one of the fractions, together state the comparison of the fractions ( $<$ ,  $>$ ,  $=$ ) and use the boards to prove their statement.

6th through 8th graders using Geoboards can be provided an opportunity to investigate some of the following applicable content standards:

- ◆ properties of shapes by creating a proof of their responses, and
- ◆ discovery and confirmation of area algorithms.
- ◆ Bonus: Tangible Geoboards versus a digital version allows this adolescent group of children who are growing into their bodies once again, the opportunity to work on fine motor skills exactly as they needed to do in Kindergarten.

IDEA: Provide students with simple questions such as, "Is it possible to create a triangle with an area of 3 and perimeter of 3?" and let them work individually or in groups to prove their answer. Providing them with an opportunity to first use a board and then recall any applicable formulas or algorithms they may have previously learned, creates a simple way to make geometry formulas unforgettable.

9th through 12th grade students can use Geoboards to work on some of the following algebraic skills:

- ◆ creating lines,
- ◆ examining slope, and
- ◆ solving systems of linear equations.

IDEA: With students working individually, ask students to represent positive and negative slopes, represent a line given the equation, represent a line and determine slope given two points, or just play a good old fashioned game of Battleship!

## References

National Council of Teachers of Mathematics. (2014). *Principles to actions: Ensuring mathematical success for all*. Reston, VA: NCTM, National Council of Teachers of Mathematics.

# ACHIEVING MEASUREMENT SENSE: PROCESS & SEQUENCING

by R. Michael Krach

## Abstract

A discussion of the types of activities that must be addressed by teachers and learned by students, and the order in which students should experience those activities, to ensure students will develop fundamental measurement sense.

According to the National Council of Teachers of Mathematics (2000), the Common Core State Standards for Mathematics (2010), and the author's 45 years of teaching experience, K-5 instructional programs in mathematics should enable all students to understand and utilize the process of measurement in order to help achieve "measurement sense". In addition, by including measurement as a Key Concept in grades K-5, the South Carolina College- and Career- Ready Standards in Mathematics (2015) consider the process of measurement as a fundamental process for all students to master and apply. When creating measurement activities for elementary school students, the teacher must keep in mind the following "process of measurement" when helping their students develop measurement sense:

- ◆ **Select an attribute to be measured.** A selected object can be measured in many different ways. For example, a coffee mug can be measured by its height, width, circumference, volume, surface area, and/or mass. The attribute being measured may be dependent upon the topic(s) being studied by the students (at any given time).
- ◆ **Select an appropriate unit of measure and compare it to the object being measured.** Every measurement is a comparison with a selected appropriate unit, using either a standard or a non-standard unit (for example, selecting the kilogram to measure the mass of a person or a paperclip to measure the width of a classroom table).
- ◆ **Count the number of units that it takes to "cover" the object being measured – that number is the measure of the chosen item.** The larger the unit, the fewer copies of the unit are needed to measure the selected item. Every measurement is an approximation. It is rarely the case that when measuring an object, a whole number of units will "cover" the object. Even when a measurement appears to be exact, using a more precise tool will usually indicate that this is not true. Therefore, the creation of "subunits" may be necessary.
- ◆ If appropriate, **a measuring tool can/should be constructed by the students**, using multiple copies of the selected unit. For example, students could create their own rulers using cash register tape as the medium and a paper clip as the unit.
- ◆ Lastly, if appropriate, **an indirect way to measure should be pursued and established.** In other words, can a relationship or formula be created that can be used to measure the chosen attribute indirectly? For example, can a formula, relating the base and height of a rectangle, be created to compute the area of a rectangle, without directly measuring the rectangle?

The following general activities and ones like them can and should be used to help elementary students develop measurement sense.

### **Premeasurement Activities**

The first measurement activities with elementary school students should involve making perceptual comparisons of two objects. For example, comparing the length of two pencils, the mass of two containers, the area of two towels, and/or the volume of two containers (by emptying the entire contents of the smaller container into the larger container). Most students are not ready to move on to more formal measurement activities until they are comfortable with these informal comparison activities.

The next activities should involve seriating (or ordering) a set of objects. Attributes, such as length, width, height, volume, and mass, should be used. At this stage of the measurement continuum (premeasurement activities), it is critical that elementary school students interact with objects using their senses and without the use of numbers.

### **Early Measurement Activities**

Use a variety of different, but familiar, objects as units, referred to as non-standard units. Students determine the length of an object by placing as many copies of a chosen unit (for example, a paper clip) as needed along the object and then counting the number of copies. The resulting number is the length of the object.

When measuring actual objects, elementary school students will quickly see that a whole number of copies of a selected unit will not always fit the object being measured. When this happens (and it will!), terminology, such as “almost 3”, “a little more than 5”, or “about  $1\frac{1}{2}$ ”, should be encouraged by the teacher when suggested by the students. For obvious reasons, measuring actual objects is a far superior activity than using drawings or pictures.

Students should have measurement experiences involving additional attributes besides length. For example, area could be investigated by using pattern pieces or sheets of paper to cover a tabletop or floor. By using these non-standard units, students will discover that regions with different shapes may have the same area. Perhaps an activity that can be conducted prior to an area activity might be a tessellation activity using five or six regular figures. A discussion of which figures tessellate a plane and the reason why would be very beneficial to students prior to a study of area. In addition, an activity, using geoboards and/or graph paper, can be presented, where the students can discover that area and perimeter are not directly related. Capacity can be explored by repeatedly emptying a container (for example, a 12-ounce Coke can) into another container, whose capacity is being measured. The students should first estimate the number of times it would take emptying the “unit can” into the container being measured and then count the actual number. The students can observe that containers that appear to have different capacities actually hold the same amount (conservation of volume). Mass /weight should first be explored by having the students lifting objects and, therefore, comparing the heaviness of different objects. After this informal activity, a balance scale should be used by the students to more formally compare the mass/weight of different objects.

Early measurement activities with non-standard units of measurement should be provided for the students. By initially using non-standard units of measurement, students will understand the need for standard units of measurement. For example, a person wants to purchase some carpet and simply uses her stride to “measure” the length and width of the room. Since this type of measuring is not very accurate and the carpet needs to

be purchased using (hard earned!) money, a standard unit of measurement must be used to make communication among the purchaser, retailer, and manufacturer possible and accurate.

### Standard Units of Measurement

Once students have a well-developed sense of measurement as a comparison with a selected unit, they are ready to measure objects with standard units. Previous activities with non-standard units of measurement along with a discussion of the need for the communication of measurements should provide motivation for the adoption and use of standard units of measurement.

The first step should be for students to realize the actual size of the units that have been accepted as standard by society (for example, the meter, mile, pound, kilogram, etc.). Students should be provided with activities in which they must relate the standard unit to a familiar object. For example, a student should be able to choose an object that measures about one centimeter from a list of 3 or 4 objects. Students should also be provided activities where they must select an appropriate unit(s) to measure a given object (and be required to justify their selection). For example, when asked to measure the height of a door way, which standard unit of measurement would be most appropriate to use? A discussion of “Could more than one appropriate unit of measurement be selected for this task?” should also be encouraged either by the teacher or by the students.

Estimating the measurement of an object should be a major component of the process of measurement. In fact, estimation should precede the actual measurement process (with or without the use of a standard unit of measurement). The more students practice making estimates, the better their estimating skills will become.

### Using a Measurement Tool

Measurement tools are devices that make a measurement activity easier and more accurate. Recall that the students should make an estimate prior to actually measuring the selected object. The reasons to do this are to develop the skill in making estimates and to maintain a sense of the relative sizes of the chosen units of measure. At this time, the teacher may wish to emphasize the proper technique of measuring. For example, if a ruler needs to be moved, how carefully must this be done? Does it matter if a ruler is worn at the edges or starts at 1 inch instead of 0 inches? Does it matter how you hold a measuring cup while filling it? The teacher also needs to make sure that the students realize that the accuracy of a measurement depends on the capabilities of the tool as well as how carefully the tool is used. As previously stated, students should be encouraged to create their own measurement tools, using both non-standard and standard units of measurement.

### Measuring Indirectly

This involves the creation and use of a formula or relationship in order to calculate the measurement without actually measuring the object. Activities must be provided to guide students to discover the formula prior to using it. For example, the use of the cardboard portion of toilet paper rolls can illustrate for students that the area formulas for rectangles and parallelograms are the same. The use of geostrips to derive the area formula of triangles can be done quite effectively after a study of transformational geometry. Using a paper plate cut into eight congruent sectors to create and justify the

area formula of a circle can also be an enlightening activity for students.



Geostrips – from [teaching.com.au](http://teaching.com.au)

## Conclusion

The process of measurement is a fundamental skill for all students to learn and apply in their daily lives. Although measurement is clearly important in mathematics and science, elementary school students often lack both procedural and conceptual competence in the area of measurement sense. Studies have shown that many students perform far below the mean on measurement items (Ginsburg et al. 2005). In addition, low performance on measurement items on the National Assessment of Education Progress provides additional evidence that measurement concepts and skills are difficult for students to grasp (Battista 2007, 2012). This paper was written to help teachers to “see” the process and to provide guidance in asking appropriate questions and in preparing/selecting activities for students to complete. Without experiencing these “process and sequencing” activities, many students could have difficulty developing a deep understanding of the process of measurement and, therefore, measurement sense.

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# QR CODES: ARE THEY ACTUALLY USEFUL IN THE MATHEMATICS CLASSROOM?

by Jennifer Zakrzewski

## Abstract

QR codes are often used within the classroom; however, it is unclear how they fit in the mathematics classroom. Within this article discover what QR codes are, practical applications for the mathematics classroom, and how to design QR codes. In addition, tips and tricks are included to help you be as successful as possible while increasing student motivation and decreasing unnecessary student questions.

QR codes have been a trend in education as of late. However, it can often be difficult to determine how to incorporate new technological trends into the mathematics classroom. While others are excited about the new potential, math educators are wondering where they fit in the mix. Well, QR codes are useful in mathematics and very easy to develop.

## The Research

According to Marzano and Pickering (2011), using game-like activities in the classroom increases student engagement. Since student engagement is something we are all after, this seems enticing, but how does this tie into QR codes? Let's back up a moment to look at what a QR code can do. QR codes are matrix barcodes (Figure 1) that can hold a range of information. Some of these items include: posters, flyers, library catalogs, and Moodle print outs (So, 2011). However, this is just the tip of the iceberg.



Figure 1

The best part about QR codes is that they can be used for learning on the go (Durak, et al., 2016). So, we might use QR codes to display information around the school building to help learners interact with the material (So, 2011). Students are more likely to engage with this information as it is interactive and allows them to connect with infor-

mation in an easily accessible manner (Durak, et al., 2016). Lastly, students are motivated by QR codes because they are easy to use and incorporate technology (Durak, et al., 2016).

To access a QR code you use your camera on your hand-held device, such as an iPad, tablet, or cell phone, and the scan connects you to the content. When considering the classroom, QR codes can be used to: link learners to websites, maps of the school, or dynamic assessments; create a treasure hunt; develop a green classroom; highlight exemplary work; provide extensions for assignments; enhance knowledge; extend learning outside the classroom; and create digital portfolios (Sharma, 2013).

## Implementation Ideas

Still not quite convinced that QR codes can be helpful? Before you make up your mind, let's discuss several methods for using QR codes specifically in the mathematics classroom.

An under-utilized method of using QR codes in the mathematics classroom is in centers, also called stations. Often, centers are used in elementary grades, but are used less often in middle school and rarely appear in high school classrooms. However, centers can be a beneficial way for students to practice mathematical tasks and provide the teacher with time to work with struggling students.

Centers can be used in several ways, but one suggestion is to attach QR codes to games students might play based on their areas of weakness. It takes but a few minutes to create QR codes, and you can provide students with several options to work on various skills. Another option would be to provide answers for the problems in a center using the QR codes. This allows students to check their work in real time, while you might be working with a small group on something else. In addition, videos can be incorporated. For example, if a student is struggling with a particular concept, maybe they scan a QR code that brings them to a video from Khan Academy to support them in their struggle, so again the teacher can work with a group uninterrupted.

Many students love to solve puzzles. Consider developing a cryptogram (Figure 2) with QR codes. Choose the problems or find a worksheet that you like, and connect each solution to a letter and develop a phrase at the bottom of the worksheet. When students answer a question, they look for the number answer around the room. On the sheet there will be a QR code. When they scan the QR code for the number solution it will connect them to a letter to fill in the puzzle clues. While I realize this is not terribly different from a cryptogram puzzle, students will enjoy being able to get up and walk around the room and use the technology as part of their assignment.

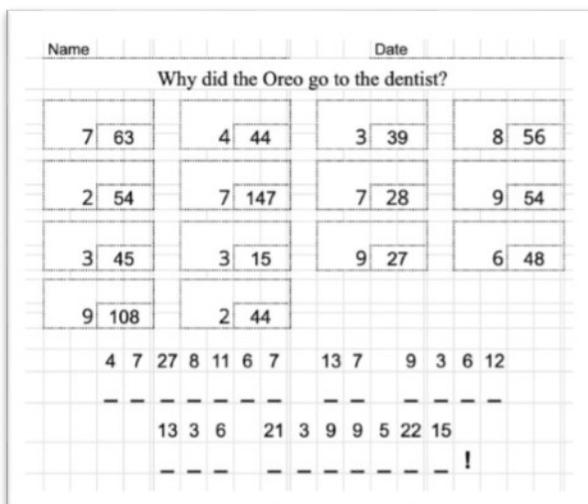


Figure 2

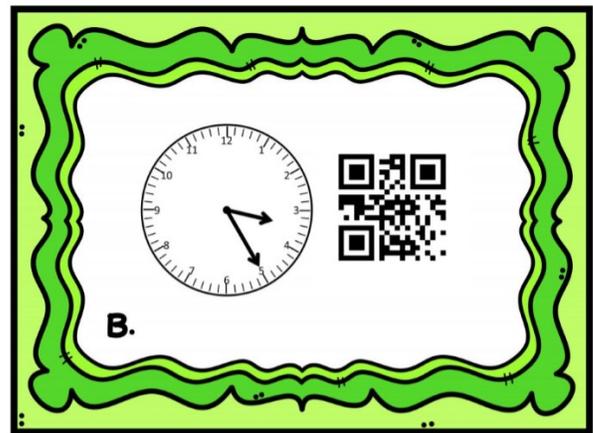


Figure 3

Many of us have used task cards (Figure 3) in the classroom before, but now we can incorporate the QR codes. This is similar to the cryptogram in some ways. Students start at one task card. When they find the solution, they scan the QR code to check if their answer is correct. This one may take some coaching for the older grades so they actually work out the problems; or you can have the solutions in a separate location so that the students need to finish all of the task cards before they check their answers.

Maybe you want to start smaller. Consider setting up a QR code for students to check homework. The QR code can be attached to an answer key. This way students can check answers when they come into the classroom and you can just go over problems that they have questions about. QR codes also work well if you use guided notes with your students. If a student is absent, have a QR code on the wall with the guided notes filled-in from the previous day. This allows the students to gather their work quickly and efficiently so you can get going with class.

What if you are absent from school? This is always a dilemma for teachers, but technology is making it easier to keep the learning going. Connect a QR code to an answer key for the homework. Have another code for tonight's homework assignment. And, you can connect a QR code to a video about the assignment for today and provide them the assignment. The sub then just

needs one sheet of paper with the QR code (or maybe a few copies) so students can scan and be on their way.

Many teachers have word walls in their classrooms (Figure 4). Try enhancing your word wall by attaching a QR code to the word that connects to a definition on the web. This is a great way to help students be self-sufficient in your classroom. Or, what about your early finishers? They are wonderful but sometimes cause anxiety if you have to create another assignment for them on a moment's notice. Create some QR codes that connect to math games to practice skills. Utilizing these simple ideas allows for the students to be more autonomous in their learning and gives you more freedom and time to focus on the students who require your attention at that time.



Figure 4

### How Are QR Codes Made?

By now I hope you are convinced that QR codes have a place in the mathematics classroom. However, you might be wondering how to develop these amazing pieces of technology. Well, let me tell you from experience that it is not challenging, but you need to use a critical eye. If you Google “QR Code Generator” you will be provided with several options. However, some of the options are free and some are not. So, you might Google “Free QR Code Generator.” This will also provide several options, but be wary.

I made the mistake of using a site that I thought was amazing. It had a version that required payment, but I didn't think I would need it. Oh how wrong I was in that regard. After developing my QR codes, I took them out a month later to use. The QR codes were no longer valid unless I paid for

the subscription. Frustration and panic quickly set in as I tried to determine a way to salvage my lesson. Luckily, I have found a site that does not allow QR codes to “expire.” The site <https://www.qrstuff.com/> is an amazing site. It does have a version for cost, but you can do most of what you need to do with the free version. This site allows you to create QR codes for websites and YouTube videos for free. That's really all you need, especially if you use a cloud-based server such as Google Drive. Any homework answer keys, guided notes, etc., can be stored in Google Drive and the share link can be used to set up the QR code.

My advice for those just getting started is start small. Use a website that you want kids to utilize in class. Copy the link and paste it into the box on qrstuff.com. A QR code will generate. Save the QR code or take a screenshot. Place the QR code in a Word document, presentation, etc., and share with your students. They can use their device's camera to scan the QR code and access the website.

### Some Warnings about Devices

Devices can be tricky when using QR codes. If students have a smart phone it is easy to scan the QR code. Most mobile devices allow you to scan from the camera app to connect to the content. However, some mobile devices require an app specifically for QR codes. Typically, you can find a free app for students to use. If your students have tablets, the app should work in a similar fashion. Students who have a Chromebook or other type of computer need to have a camera on their device. In addition, with any school devices you will want to check that the scanning capability has not been blocked or deactivated. Your administration and/or instructional technologist may be able to support you in getting an app or finding a method of scanning that works with the school-provided devices.

## Final Thoughts

I hope I have piqued your interest regarding QR codes and how they can be useful in the mathematics classroom. You may have questions as you start to implement their use in your classroom. Contact your instructional technologist (they have a plethora of ideas) or see the attached presentation from the 2019 SCCTM Conference (Figure 5). The presentation includes the methods discussed in this paper, and links are incorporated for creating QR codes and accessing various activities for the mathematics classroom.

Now, go, QR, and be merry!



Figure 5

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