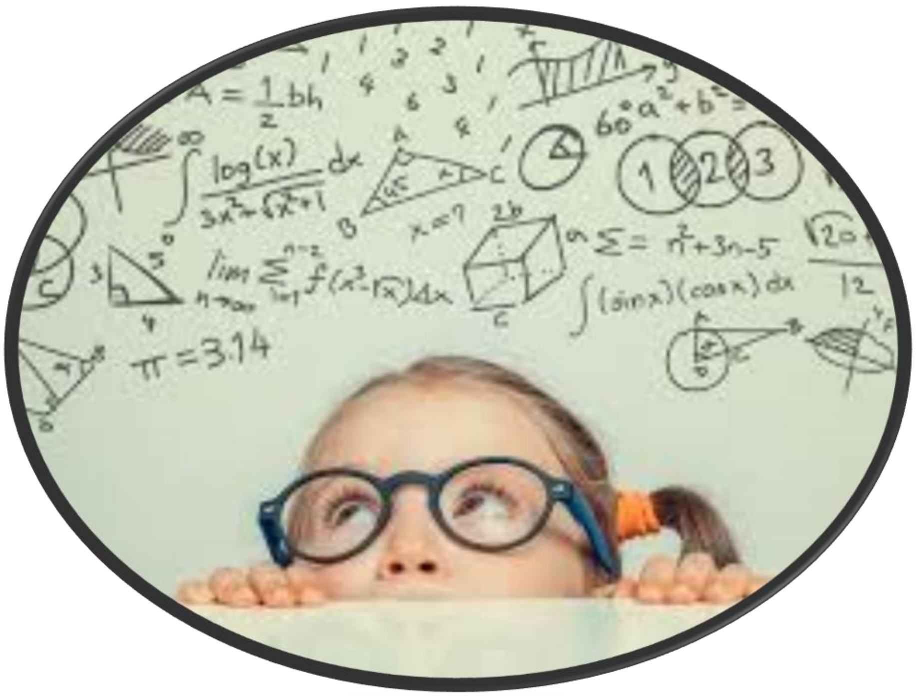
Volume 43 | Number 1 | August 2022

THE MATHMATE

The Official Journal of the
South Carolina Council of Teachers of Mathematics



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THE MATHMATE

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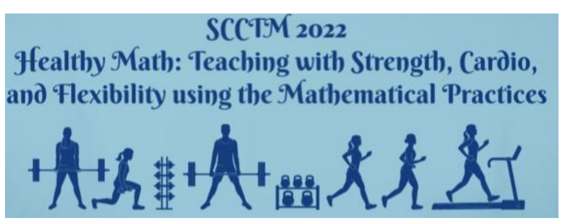

THE MATHMATE

Mission Statement: The mission of The MathMate is to feature articles about innovative mathematical classroom practices, important and timely educational issues, pedagogical methods, theoretical findings, significant mathematical ideas, and hands-on classroom activities and make this information accessible to students, educators, and administrators.

Submission Requirements: All submission are to be emailed to mathmate@scctmconference.org as attachments along with a completed Submission Coversheet. Submitted files must be saved as MSWord or PDF files. Pictures and diagrams must be saved as separate files and appropriately labeled. Authors are asked to not submit the same article to another publication while it is under review for The MathMate.

Submission Deadlines: The MathMate is published three times per year. Submissions received by November 1 will be considered for the January issue, March 1 for the May issue, and July 1 for the September issue.

South Carolina Recertification Credit: According to the SC Department of Education Renewal Credit matrix, the primary author of a peer reviewed journal article can earn 60 renewal credits.

Subscriptions: Active members of SCCTM receive online subscriptions to The MathMate as part of their membership.

Correspondence: All correspondence regarding The MathMate should be sent to mathmate@scctmconference.org.

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MESSAGE FROM SCCTM PRESIDENT

Dear Members,

As we near the end of a unique and challenging year in education, I hope that you will take the time to reflect on all that you have accomplished. I am impressed and motivated daily by the hard work and dedication of my colleagues across the state. From monitoring and implementing all covid protocols to changing the entire structure of the educational platform, you all did it with very little help. Educators are the epitome of resilience.

It is our goal at SCCTM to help support teachers and promote mathematics education by offering a researched-based, professional publication. The board of SCCTM would like to thank all authors for sharing their knowledge by writing for The MathMate. We would also like to thank Leigh Martin for serving as the editor for this edition. We hope you will consider contributing to this publication by sharing engaging lessons, interesting activities, or classroom strategies. Your work can benefit other mathematics educators in our state.

Our annual conference will be in Greenville, November 17-18, 2022. I hope that you will take advantage of the discounts and register early. Visit our website for more information: http://www.scctm.org/.

Sincerely,

Alisa Hobgood, President

Message from the Editor

This issue of *The MathMate* incorporates practitioner articles, including lesson plans and activities, with peer-reviewed research-based articles. The South Carolina Council of Teachers of Mathematics is excited to transition *The MathMate* to a hybrid journal that includes both types of articles. So, if you have lesson ideas and have been hesitant to submit those ideas in the past, now is the time to consider sharing your ideas and activities with your peers in *The MathMate*!

The SCCTM Board encourages all members to consider submitting an article, lesson plan, activity, or lesson strategies to be included in the September issue of *The MathMate*. In addition to receiving recertification credit, this is a wonderful way to share ideas with peers and learn from others. We hope you will consider submitting your ideas to be included. All submissions should be shared via email, mathmate@scctmconfrence.org, by December 1.

Thank you,

Leigh Martin, Editor of *The MathMate*

ANNOUNCEMENTS

Award Nomination Deadlines:

Outstanding Contribution to Mathematics Education Award
Nomination deadline: July 15
scctm.org/Awards

Richard W. Riley Award
Nomination deadline: July 15
scctm.org/Awards

Scholarship Deadlines:

Preservice Scholarship
Applications deadline: September 15
scctm.org/scholarships

Educator's Scholarship
Application deadline: September 15
scctm.org/scholarships

Membership News:

Renew your NCTM membership online and designate *South Carolina Council of Teachers of Mathematics* for the affiliate rebate.

NCTM NATIONAL COUNCIL OF TEACHERS OF MATHEMATICS

2022 SCCTM Fall Conference: Keynote Speakers

Dr. Andrew "Andy" Tyminski is an Associate Professor of mathematics and mathematics education at Clemson University. He has worked extensively developing elementary mathematics teachers' content and pedagogy through coursework and professional development activities. His research interests include the effect of learning trajectories in content courses on prospective teacher noticing, the content of mathematics methods courses, and teachers' responsive problem-posing practices. He has been a Co-PI on both federal (NSF) and state-level (MSP) grants. He has recently published in the *Journal of Mathematics Teacher Education*, *Teaching and Teacher Education*, and *Mathematics Teacher Education and Development*. He is a co-editor of the AMTE Professional Series book, *Building Support for Scholarly Practices in Mathematics Methods* (Kastberg et al., 2018). Prior to his career in higher education, he was a middle school mathematics teacher in Virginia Beach, VA and at the American School of the Hague in the Netherlands.

Zachary Champagne has been involved in mathematics education for over 20 years. He believes that each student has important mathematical ideas and works to share his passion and love for mathematics with teachers around the country. Zak is currently serving as Lead Teacher at The Discover School in Jacksonville, FL where he teachers third and fourth grade students. He has received many state and national awards for excellence in teaching, including the Presidential Award for Excellence in Mathematics and Science Teaching (PAEMST), Duval County Teacher of the Year, Finalist for Florida Teacher of the Year, and the Kenneth Kidd Mathematics Educator of the Year award.

Kevin Dykema has been an 8th grade math teacher in southwest Michigan for over 25 years and is currently serving as President-Elect for the National Council of Teachers of Mathematics before starting a 2-year term as President in October of 2022. He is the co-author of *Productive Math Struggle*. He also conducts many professional development sessions throughout the United States and loves working with others to help improve mathematics education for each and every student.

Dr. Bernard E. Frost is currently the Executive Director of Curriculum and Instruction for Spartanburg School District Seven. Dr. Frost also serves as the Southern 1 Regional Director of NCTM Membership and Affiliate Committee, is a board member of SC Learning Forward, the Past-President of South Carolina Leaders of Mathematics Education, past Director of Teacher Quality and Staff Development for Spartanburg School District Two, and most recently the 2020 recipient of the SCCTM Outstanding Contribution to Mathematics Award. With over 17 years of teaching experience and conducting professional development, Dr. Frost's passion for education is evident in his willingness to put forth 100% in developing professional development opportunities that assist educators in their ongoing process of improving instructional practices that impact student achievement. As a consummate research and practitioner in the field of education, Dr. Frost focuses on ways to improve instructional practices and student achievement. He continuously researches new strategies to meet the needs of administrators, instructional coaches, teachers, and students. Frost says, "Inspiring other educators provides great insight on the future of education and allows me to have an impact on it."

If you would like your announcement to appear in the next issue of The MathMate, please email all information to mathmate@scctmconference.org. Announcements will be published at the discretion of The MathMate Editorial Board.

SCCTM Outstanding Contributions to Mathematics Education Award

Marc Drews

EdVenture's Marc Drews was honored by the South Carolina Council of Teachers of Mathematics with the 2021 Outstanding Contributions to Mathematics Education Award at their organization's annual conference on November 11 at the Columbia Metropolitan Conference Center.

Drews has been a member of the EdVenture team since the summer of 2012 and currently serves as the museum's Vice President of Community and Government Relations. He has been a longtime advocate for STEAM education and serves a member of the advisory boards of the SC Coalition for Mathematics and Science and the SC Organization of Rural Schools.

This is the second time Drews has been recognized by the organization with the award, having been a recipient in 1993. Fifty-six math educators have received the award since 1982 and, he is one of three educators who have been repeat honorees. He is the first recipient who represents an informal education organization.

His work extends beyond the world of mathematics education, having served as a program co-coordinator of the SC Arts Education Association's annual conferences in 2010 and 2011, along with his son, Josh. He was recognized by the SC International Baccalaureate Schools in 2008 for his work promoting international education and by the SC Science Council as the recipient of the 2015 Catalyst Award for contributions to science education.

Drews has served the SCCTM as the editor of The MathMate for eleven years and, most recently as its 42nd president and chaired last year's online program that included a series of monthly gatherings with keynote speakers who he charged to inspire and lift the spirits of teachers during the height of the pandemic.

> *In the face of overwhelming challenges during the last two years, Marc Drews has been a positive influence in the lives of many math educators and children.*
>
> –Dr. Ryan M. Higgins
> SCCTM President 2019-2021

Drews joined EdVenture after a career that began over forty-five years ago in Charleston County, where he taught high school students to appreciate algebra and geometry. In 1987, he accepted a position as a math consultant with the SC Department of Education, where he worked in a variety of positions, retiring on Pi Day 2008.

While at the SCDE, in January 1996, he was named the principal investigator of the NSF grant that helped create the state's infrastructure, known as the regional Hubs, designed to support science and mathematics education. As the director of the South Carolina Statewide Systemic Initiative, he led the efforts of the Hub community and coordinated the work of the Governor's Math and Science Advisory Board. He served as the state superintendent's designee to the Governor's School for Science and Mathematics' Board of Trustees for eleven years. He also managed statewide programs such as the K-5 Enhancement (Lottery), the AP and IB programs, and financial literacy.

About the Author

Drews proudly identifies his greatest roles as being a husband, a father of three adult children and their spouses, a grandfather of seven enjoyable and entertaining grandchildren, and the human friend to their rescue pet canine, Lilly.

Draw a mathematician: how female secondary students view mathematics and mathematicians

by Sandra Davis Trowell and Denise T. Reid

Abstract

Female secondary mathematics students were asked to Draw a Mathematician. The use of drawings and brief narratives were examined to make sense of current students' beliefs and attitudes about mathematics and mathematicians and thereby, assess the potential impact upon future choices and decisions. It is imperative that students see mathematics and mathematicians outside of a mathematics classroom and understand the importance of mathematics in our world. As we strive to achieve equity in the study of mathematics and other STEM fields, these findings can assist in making choices in K-12 classrooms to positively impact mathematics classrooms.

The NCTM document *Principles to Actions: Ensuring Mathematical Success for All* (NCTM, 2014) discusses the strengths and weaknesses in mathematics education as we continue to take action so that all students have the opportunity to become confident and capable in their mathematics learning. In the final section of this document looking toward the future, the authors state that it is desired that "students believe that mathematics makes sense, appreciate mathematics as a field of study, and are willing to consider the possibilities for further studies in mathematics or mathematics-based fields" (NCTM, 2014, p. 109). In an effort to support this endeavor and encourage secondary female students to further their studies in STEM (Science, Technology, Engineering, and Mathematics), an all-day event is held at a regional university in the southeastern United States that focuses upon encouraging the participants to continue their studies beyond high school in STEM fields. This paper examines the beliefs and attitudes of these secondary students toward mathematics and what it means to be mathematicians.

Theoretical Framework and Related Literature

According to the Bureau of Labor Statistics, "overall employment of mathematicians and statisticians is projected to grow 33 percent from 2016 to 2026, much faster than the average for all occupations" (Bureau of Labor Statistics, 2018). The Bureau of Labor Statistics expects as more data becomes digitally stored there will be an increased need for mathematicians to analyze this information and data. In addition, as our population ages, there will be a greater need for mathematicians in scientific research and pharmaceutical development. Not only does the U.S. Bureau of Labor see a growth in careers of mathematicians and statisticians but they predict above average growth in other STEM occupations. Between May 2009 and May 2015, employment in STEM occupations grew by 10.5 percent while non-STEM occupations grew by a smaller 5.2 percent (Fayer, Lacey, & Watson, 2017). The U. S. Bureau of Labor predicts that growth and need will continue in STEM areas with the fastest growing group from 2014-2024 to be the mathematical science occupations group at 28.2 percent with over 99 percent of STEM employment requiring some type of postsecondary education (Fayer, Lacey, & Watson, 2017). The National Girls Collaborative Project (2018) reports that females continue to be underrepresented in the science and engineering workforce. While constituting 50 percent of the overall workforce, they make up only 28 percent of the science and engineering workforce. This project reported data from the National Science Board that showed that females represent only 26 percent of those working in the mathematical sciences and 15 percent in engineering (2018).

Student's beliefs about mathematics and mathematicians will contribute to their desire and willingness to pursue further studies in mathematics or any study that requires further study of mathematics. If students perceive mathematics in a negative way and fail to continue in their studies, there will be a shortage of qualified mathematicians and other STEM occupations. Rock and Shaw (2000) recognize that the importance of probing students' beliefs about mathematics and mathematicians, so that we "might alleviate misconceptions and facilitate and broaden children's thinking about their roles as future mathematicians" (p. 550). Finson (2002) examines what we have learned from fifty years of Draw-A-Scientist and notes that "having some foreknowledge of students' perceptions of scientists may be important to educators if they are to effectively and positively impact them through instruction" (p. 335). This could also transfer to students' perceptions of mathematicians including their willingness to pursue future studies of mathematics. Students must recognize mathematics as a dynamic and engaging activity in order to be encouraged to pursue further studies in mathematics.

Draw-A-Scientist and subsequently Draw-A-Mathematician have been used to investigate the beliefs and attitudes of students toward science and then mathematics (Reinisch, Krell, Hergert, Gogolin, & Kruger, 2017; Finson, 2002; Picker, & Berry, 2000; Mille, Noll, Eagly, & Uttal, 2018). Henrion (1997) suggests that imagery plays a major role into one's beliefs and "it reveals underlying beliefs, assumptions, and expectations. Moreover, imagery not only reflects but affects who goes into mathematics" (as cited in Picker, & Berry, 2000). Students in the secondary grades typically have firmed their beliefs concerning what possible career paths they see as feasible for them. This study examined the beliefs of a group of female secondary mathematics students by asking them to Draw-A-Mathematician.

Methodology

Each spring a regional southeastern university invites all public and private schools within the university's region, as well as those beyond to participate in an annual Sonia Kovalevski Mathematics Day. During this daylong event, the goal is to encourage these students to continue their studies in mathematics and related fields by hosting career speakers and providing hands-on STEM workshops. The day is coordinated by a mathematician and a mathematics educator with whom the students interact throughout the day. Each school is asked to bring up to six female participants and many bring what are considered their stronger mathematics students.

The eighty-three students who participated in the most recent Mathematics Day were asked to Draw-A-Mathematician. Each student was given a sheet of paper with the instructions, "In the box below, draw what you think a mathematician looks like while at work." In addition, on the lower third of the paper, they were asked to "Describe what the mathematician is doing in the picture." The study was primarily qualitative with the researchers analyzing the drawing and descriptions to look for patterns and episodes to illustrate claims and assertions. The researchers looked for common and recurring themes or incidents.

Findings and Discussion

In examining the drawing and narratives, it was found that 55 (~66%) of the drawings depicted a female mathematician, 21 (~25%) drew a male mathematician, and 7 (~8%) were indeterminable. So a majority of these female secondary students thought of a female when drawing a mathematician. This may not be surprising as this was an all-female group and according to Dickson, Saylor, and Finch (2002) when subjects are asked to draw a person, in general, they typically draw a person of their same gender (Finson, 2002). Another factor that may impact their tendency to draw a female is that the Sonia Kovalevski Mathematics Day was sponsored and presented by two females from the hosting university's mathematics department. In addition, of the twenty-three teachers who attended and brought their students 20 (~87%) were female with only 3 (~13%) male teacher participants. As the students were surrounded by females, it possibly impacted their decision to draw a female.

In addition, many of the images drawn had illustrations that pointed to the students viewing a mathematician as being synonymous with mathematics teachers (see Figure 1 for examples). This again may also impact their decision to draw female mathematicians given that most of the mathematics teachers in attendance were female. In their drawings, 46 (~55%) showed images that pointed to the mathematician being in a classroom environment. There were 31 (~37%) that showed no indication of their mathematician being in a classroom while 6 (~7%) were indeterminable concerning their location.

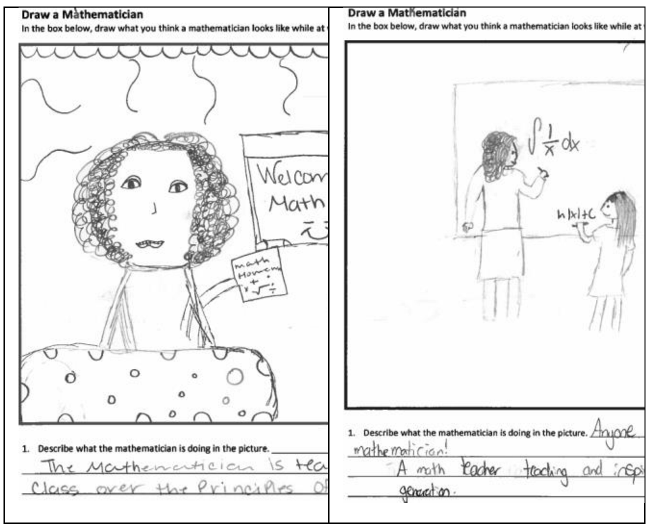

***Figure 1**: Examples of students viewing mathematicians and mathematics teachers*

Given that 37% thought of mathematicians outside of a classroom, that is a positive development for the future and continued study of mathematics. Hammond, in 1978, referred to mathematics as an invisible culture as many students, and perhaps the public at large, do not know what a mathematician actually does (Picker & Berry 2000, p. 71). The participating students who drew a mathematician outside of a classroom environment demonstrated recognition of a mathematician that exists beyond a mathematics classroom (see Figure 2 for examples). Many times, their choices for the mathematician showed them working alone with some using a computer.

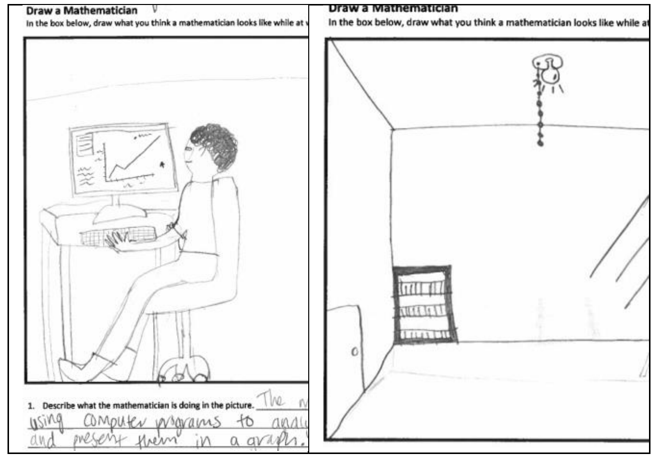

***Figure 2**: Examples of students viewing mathematicians outside a classroom*

The mathematicians, both teaching and nonteaching, sketched by these students were typically shown to be standing whether at the front of a classroom or at a board solving or demonstrating. One might interpret this to assume that the students viewed mathematicians to be active. Sixty-two (~75%) illustrated their mathematicians in a standing position while 19 (~23%) portrayed their mathematicians sitting, and 10 (~12%) were indiscernible as to their position. Again, this may be reflective of the action of their current and former teachers. Most of the students could be construed as describing or expressing mathematics or mathematicians in a positive light, but 11 (~13%) of the students used words and/or descriptions that could be interpreted in a negative manner. Students mentioned words such as difficult, hard, or confused or the expressions of the mathematicians appeared unhappy or irritated. Some students whose drawings showed classrooms, expressed frustration in the depicted teacher's role as she appeared to be frustrated or upset with a student.

A chalkboard or whiteboard was seen as an essential item for an overwhelming majority of the students. Sometimes these boards were in classroom environments, but other times they appeared to be in an office or other workspace with the mathematician solving problems. Only 16 (~19%) of the eighty-three students did not draw a chalkboard or a whiteboard, yet 67 (~81%) drew a board for their mathematician. At times, the boards appeared to be used for demonstration purposes such as in a classroom but other times they appeared to provide vast workspaces for the mathematician.

In the analysis of these sketches, there were a few celebrity presences that did appear (see Figure 3 for examples). One student drew Albert Einstein and included a quote that she attributed to Einstein, “We cannot solve our problems with the same thinking we used when we created them.” Even though Einstein is considered a scientist, there are those who will default to Einstein as the classic nerd or awkward geek with characteristics that may also be attributed to typical mathematicians. Another student depicted someone who appears to be Alan Turing. The student simply draws a board with the word “math” with bubble beside it exclaiming, “Go Allies” and “Boo Hitler.” She further describes her picture stating that, “Alan is my main dude.” She then describes him as working on code used by the Germans and henceforth cracks their code. At some point this student became familiar with Alan Turing and knows quite a bit about this mathematician and cryptologist.

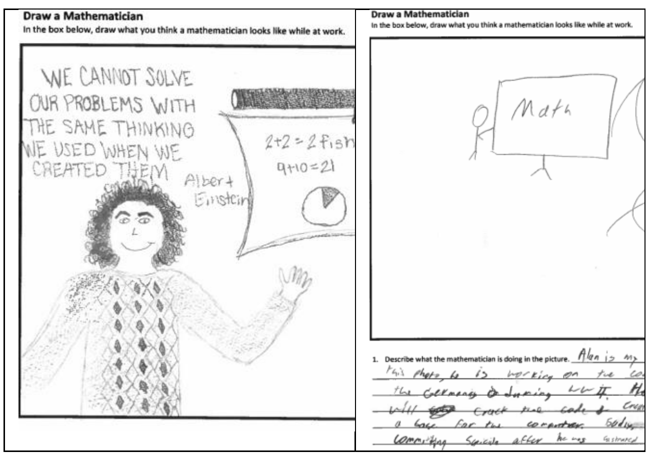

***Figure 3**: Examples of celebrity mathematicians*

To illustrate the importance of current media, five students portrayed Katherine Johnson from the recent movie *Hidden Figures* (see Figure 4 for examples). The influence of this movie was apparent in these five representations but was also apparent in other student’s ideas. There were five students whose mathematics depicted physics trajectories. In *Hidden Figures*, Katherine Johnson solves problems on a very large chalk board with a ladder for reaching the full height of the board. Five other students, besides those who specifically drew Ms. Johnson, also drew a ladder beside their boards. This movie obviously had an impression upon some of the participating students and their views concerning mathematicians and the work of mathematicians.

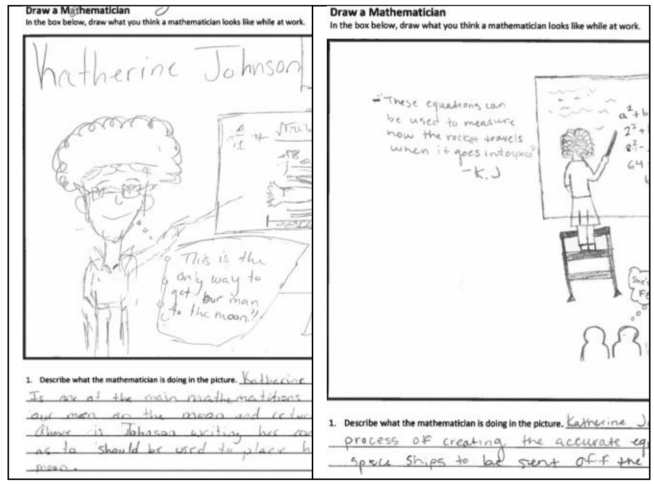

***Figure 4**: Examples of students’ representations of Katherine Johnson*

The Draw-A-Mathematician challenge is a quick and cursory glimpse into these female students’ ideas about mathematicians and what mathematicians do. Latterell and Wilson (2012) use a working description of what mathematicians do as “a mathematician creates mathematics” (p.73). In examining these submitted drawings, there were 12 (~14%) whose submission indicated a mathematician creating mathematics (see Figure 5 for examples). One student describes her mathematician saying, “She has been working on a new formula nonstop for months. She hasn’t slept in four days because she just had a breakthrough.” From these samples it appears that perhaps some of these secondary students do have a sense that mathematicians do create mathematics.

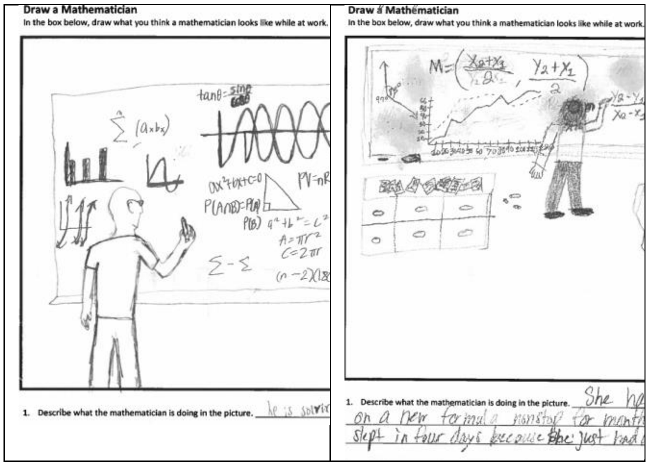

Figure 5*: Examples of students viewing mathematicians as creating mathematics*

The view of mathematicians appears to be somewhat limited for a majority of these female students. During this event and after this Draw-A-Mathematician activity, the students were then asked to share with the group the name of a female mathematician. Among the students, the only female mathematicians that were named were those portrayed in the *Hidden Figures* movie. While this movie certainly portrayed the possibility of females becoming mathematicians or scientists, this was the only example for those present. The eighty-three female student participants recognized that they were participating in a day that was focused upon mathematics and that the coordinators were female. In addition, most of the students who participated were stronger mathematics students, so their mathematics experiences had perhaps been more positive than other secondary students. It is recognized that these facts must be taken into consideration when examining their drawings.

All of these students could benefit from further experiences that provide opportunities to know what mathematicians do as well as recognizing the prospects that exist in mathematics. Educating in-service and prospective teachers in knowing what mathematicians do may also impact current and future students and their views about mathematicians and mathematics. In addition, educating in-service and prospective teachers about mathematics and its creative essence should be a focus of mathematics education. This focus could in turn have a positive impact upon K-12 mathematics classrooms and lead to mathematics classrooms that are creative and exciting learning environments. It is imperative that we continue to grow mathematicians and other STEM professionals. As Latterell and Wilson note, "Most secondary students are almost completely unable to describe what mathematicians do. It is unlikely that a student would desire to pursue a career of which they have no understanding" (2012, p.73). Urschel, a former NFL player and current Ph.D. candidate in mathematics at Massachusetts Institute of Technology, says that "math teachers should be more like football coaches" in the way that they encourage students to continue in mathematics and get them excited about this field (2019).

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About the Authors

Dr. Trowell received her Ph.D. from Florida State University and is currently a professor in the mathematics department at Valdosta State University.

Dr. Reid received her Ph.D. from Auburn University. She is a full professor of mathematics at Valdosta State University.

Storytelling as best practice using 3-act Math Tasks

By Brandie D. Maness and Deborah H. McMurtie

Abstract

Effective teachers of mathematics create purposeful learning experiences for students by presenting novel problems in authentic and meaningful contexts. Storytelling provides rich contexts for problem solving. Stories can be used to present material as a mystery, and students will be naturally inclined to "figure out" the solution, thus engaging in the process of making meaning. We argue that 3-Act Math Tasks are curiosity-inducing and challenging while making math accessible to all learners.

Introduction

How do we create a culture of engagement, curiosity, and persistence in math class? Consider the power, the order, and the wonder of the number 3. Psychology tells us that the human brain loves patterns. Our brains are hard-wired to create meaning from the patterns we see, and the number 3 is the minimum number needed to make a pattern (Clark, 2019). The "rule of three" states that a trio of events or characters is more satisfying and effective than other numbers. The Latin phrase "omne trium perfectum" means everything that comes in threes is perfect; every set of three is complete (Burns, 2013). The ancient Greek philosopher and father of mathematics Pythagoras himself believed three to be the most perfect number and was rumored to have suffered the consequences of that conviction in the form of some brutal bullying from his peers. Have we convinced you yet?

The Magic of 3

Consider the prominence of the number three in daily life: the division of a day (morning, noon, and night), three circles on a stop light, three meals a day, to name a few. We see the number three in photography's rule of thirds, the triads used to create harmonies in musical composition, and the number of events in a triathlon. Which planet from the sun is our home? That's right! The third planet from the Sun is Earth. How many times have you used the phrase lifted from America's pastime of baseball – three strikes and you're out? How many clicks of Dorothy's slippers take her home to Kansas in *The Wizard of Oz*? Of course, we cannot discuss the number three without mentioning its significance in spiritualism, symbolism, and superstition. Multiple sects of Christianity believe in the power of the Father, Son, and Holy Ghost. In symbolism, there is the trinity which is a symbol of body, mind, and spirit. Maybe you or someone you know have uttered the old wives' tale, "Death and babies come in threes." What if we consider the use of the number 3 in advertising? Spiteri (2018) points out some of the most memorable advertising slogans in our culture which just all happen to contain three words:

- Just Do It
- Finger Lickin' Good
- I'm Lovin' It

The Number Three in Storytelling

The rule of three is a common storytelling device and rhetorical technique used in poetry, songs, jokes, folktales, and fairy tales. Some of our most beloved tales include *The Three Little Pigs, Goldilocks and the Three Bears, Three Little Kittens, The Three Blind Mice,* and *The Three Billy Goats Gruff.* When young learners begin to explore story structures, they are taught the most basic elements of beginning, middle, and end. The 3-act structure is effectively used in most stories, including plays, movies, and novels (Pressfield, 2020). Act 1 sets the stage. Act 2 presents a problem or issue and builds tension. Act 3 brings about a resolution. For example, consider the story of *Rumpelstiltskin* told in three acts (Pressfield, 2020):

1. A miller brags that his daughter can spin straw into gold. The king puts her to the test, but if she can't do it, she will die.
2. Rumpelstiltskin offers to help her 3 times, but she must give him her necklace, her ring, and her firstborn child unless she can guess his name.
3. The miller's daughter marries the king, guesses Rumpelstiltskin's name, and keeps her child.

Let's take that same storytelling structure and apply it to math. This is the key to the 3-Act Math Task. The 3-Act Math Task strategy was created by Dan Meyer as an approach to mathematical thinking with a storytelling framework. As the title suggests, 3-Act Math Tasks are presented in three acts designed to build student engagement, curiosity, and wonder (Englard, 2015). In short, teachers can harness the power of 3 to help students make sense of problems and persevere in solving them – the first of eight Standards of Mathematical Practices (Common Core State Standards Initiative, 2020).

Literature Review

It is well documented that young children arrive at school with intuitive mathematical understandings (Kurz & Bartholomew, 2012; Landrum, Brakke, & McCarthy, 2019; Lemonidis & Kaiafa, 2019). Teachers can build on these understandings through experiences that allow students to explore mathematics, as opposed to memorizing isolated facts. In the traditional mathematics classroom, students are asked to memorize numbers and operational procedures. This approach is similar to the traditional phonics-based reading curriculum in which small units of sound are taught in isolation. In both mathematics and reading, "there comes a point at which memorizing small units of data must give way to the larger meanings and understandings that comprise whole word problems (mathematics) and stories (literature)" (Kurz & Bartholomew, 2012, p. 185).

Effective teachers of mathematics create purposeful learning experiences for students by presenting novel problems in authentic and meaningful contexts. Utilizing stories in teaching mathematics is an effective method to provide this context. Most students enjoy stories and respond positively to them. Storytelling is "culturally universal and likely the oldest form of teaching, allowing generations of humans to share cultural knowledge to be remembered over time" (Landrum, Brakke, & McCarthy, 2019, p. 247). Storytelling supports memory, provides motivation, and improves analytical skills. Storytelling connects the mathematical problem solving process with the process involved in understanding the content of a story. Furthermore, combining storytelling and mathematics allows students to use strengths in one cognitive area to support learning in another (Lemonidis & Kaiafa, 2019). Teachers can use stories to introduce, explain and discuss mathematical concepts in a memorable way. Integrating storytelling within mathematics instruction develops literacy skills and promotes mathematical language.

Storytelling "helps teachers to create a dynamic and interactive learning environment that promotes mathematical vocabulary" (Lemonidis & Kaiafa, 2019, p. 165). The ultimate benefit of storytelling "is the special status that stories have regarding their memorability, derived through their narrative structure and the emotional investment they elicit from their audience" (Landrum, Brakke, & McCarthy, 2019, p. 251).

Taking mathematical concepts and recasting them into narratives thus becomes a powerful pedagogical tool (Moyer, 2000). A good story can capture students' interest, increase their engagement, and spark creativity (Faruk-Islim, Ozudogru, & Sevim-Cirak, 2018; Dietiker, 2015). In addition, using stories encourages persistence, reduces anxiety, and inspires collaborative learning (Corp, 2017; Clinton & Walkington, 2019; Lemonidis & Kaiafa, 2019). Carefully selected stories allow students to see how mathematics is used in real life and how mathematical reasoning is used to solve everyday problems (Corp, 2017). In the classroom, a storyteller can present material as a mystery, and students will be naturally inclined to "figure out" the story, thus engaging in the

process of making meaning (Landrum, Brakke, & McCarthy, 2019). Stories are even more effective when they contain unusual or memorable content.
Einstein, McDaniel, and Lackey (1989) describe the "bizarreness effect." They found that introducing bizarre or unusual information results in better recall. Thus, if a teacher creates intrigue or an unusual outcome to a story, students are more likely to remember the information (Landrum, Brakke, & McCarthy, 2019). We argue that 3-Act Math Tasks tap into the bizarre and memorable. Watching a carton of eggs drop to the floor, guessing how many little ketchup bottles will fit in a big bottle, and observing candies being poured into a lightbulb-shaped container are certain to capture students' interest. 3-Act Math Tasks create an engaging hook that even our most reluctant students are unable to resist.

Stories provide rich contexts for problem solving. The importance of problem-solving in learning mathematics stems from the belief that mathematics is primarily about reasoning, as opposed to simply repeating a set of memorized, rehearsed procedures. Students acquire their understanding of mathematics and develop problem-solving skills as a result of applying their knowledge in new situations, rather than being taught skills in isolation. Mathematics requires not only computational skills but also the ability to think and reason mathematically in order to solve new problems. Problem-solving thus allows students to transfer what they have already learned to unfamiliar situations (Kurz & Bartholomew, 2012).

Providing students with authentic problems invokes inquiry and helps them actively construct their ideas about mathematics. Students acquire understandings through engagement with problems and interaction with others in these activities. They take risks, try new strategies, and give and receive feedback. They share a range of points of view and discuss different ways of solving a problem. It is through talking about problems and discussing their ideas that children construct knowledge and acquire the language to make sense of their experiences. It allows students to test mathematical boundaries, explore mathematical ideas and relationships, and think creatively (Moore-Russo, Simmons, & Tulino, 2020). Students apply, self-monitor, and adapt new mathematical knowledge to fresh situations and contexts. Moore-Russo, Simmons, & Tulino (2020) note the difference between exercises and problems: "exercises are tasks for which the solution path, even if tedious and lengthy, is known, and problems are tasks for which the solution path [is] not immediately obvious" (p. 132).

The challenge for teachers is ensuring that problems are designed to support mathematics learning and are appropriate and challenging for all students. The problems need to be difficult enough to provide a challenge but not so difficult that students cannot succeed. In 1945 Pólya published four principles of problem-solving to support teachers in helping their students. He argued that problem-solving is not linear but rather a complex, interactive process. Students move backward and forward between and across Pólya's phases as they: 1) understand and explore the problem; 2) find a strategy; 3) use the strategy to solve the problem; and 4) look back and reflect on the solution. A good starting point for teachers "to encourage mathematics creativity may occur by prompting students to pay attention to their wonderings. . . [and to] capture their ideas and build on them" (Moore-Russo, Simmons, & Tulino, 2020, p. 131).
According to Ahmed (1987), effective problems:

- Are accessible and extendable
- Allow individuals to make decisions
- Promote discussion and communication
- Encourage originality and invention
- Encourage "what if?" and "what if not?" questions
- Contain an element of surprise

To solve a challenging problem, students must engage in self-regulated learning (SRL), which theorists define as students' ability to deliberately use cognitive and metacognitive processes to achieve a learning goal (Zimmerman & Schunk, 2011). Models of SRL generally describe how students define the task, set goals, plan and enact strategies, and evaluate progress during an academic task. Munzar, Muis, Denton, & Losenno (2021) examined what happens when this process is interrupted because of an impasse: "Whether it be a gap between prior knowledge and the conceptual demands of the problem, a lack of motivation, attentional issues, or weak domain general problem-solving skills, students will likely experience an impasse when trying to solve a challenging problem" (p. 2). Confronting an impasse is an important part of the problem-solving process. Munzar, Muis, Denton, & Losenno (2021) argue that impasse-related emotions play a critical role in learning tasks; as they unfold, they provide feedback to students in terms of how well they are comprehending the content and progressing through the task. Challenging problems force students to focus on what caused the impasse, which results in deeper learning.

What is a 3-Act Math Task?

Storytelling gives us a useful framework for mathematical tasks, particularly problem solving. Most stories divide naturally into 3 acts, which can then be mapped onto math tasks (Meyer, 2011). The 3-Act Math Task was developed by Dan Meyer (2011) and can easily be adapted for K-2 students. This method fosters constructivist learning with the teacher as facilitator while the students actively engage in structured inquiry. This approach positions students as active learners, constructing their knowledge of mathematics through exploration, collaboration, and reflection.

3-Act Math Tasks are math problems that are structured in three "acts." Each act presents the audience with a new piece of information to solve the problem. Unlike most traditional math problems, the focus is more on the process as opposed to the product. This strategy, at first glance, seems like an open-ended math exercise based on a basic explanation of the strategy. However, 3-Act Math Tasks are very involved and calculated (Meyer, 2013). Act 1 serves as an easy entry point to engage learners at any ability level in the problem. This initial engagement usually consists of a brief video provocation presented to the learner. For example, students may wrestle with questions like: How long will it take the dog to pop all of the balloons? How many sugar packets are in a bottle of soda? How many drops will fit on a penny? Following this video, the facilitator elicits questions from the learners; this process is commonly referred to as "wonderings." These wonderings should be revisited with as many being answered as possible by the end of the third act. The next part of Act 1 is to make estimates: one that is too high, one that is too low, and one that might be just right. Going into Act 2, the facilitator presents more questions to guide learner thinking such as, "What information do we still need here?" More pieces of information are shared with the learners during this act usually via another video or photograph. Finally, in Act 3 a solution is revealed. What's wonderful about this act is that it is usually met with audible reactions from the audience as their own solutions are either validated or not, signaling an emotional investment.

In a university course for preservice teachers taught by one of the authors, 3-Act Math Tasks of varying grade levels have been introduced as one of the many strategies for best practices. Selected tasks were modeled for the students in the class. Engagement of the class was high with all students contributing estimates for the numbers of items relevant to the specific tasks. Students were asked to figure out how many Pringles it would take to create a stacked circle; how many 12-packs of Coke were used to create a model of the Olympic rings; and how many graham crackers are in a box. Some of the comments from students included:

- "I've never heard of this. I can't wait to try this out with the students in my practicum!"
- "I can see how struggling students could be successful with this. I've never been good in math, but I feel like I could have done this when I was a kid."
- "I had so many questions after Act 1! I was hooked. I couldn't wait to find out the solution!"

Standards and Best Practices in Math

How does the 3-Act Math Task strategy support research-based standards and best practices in mathematics? In Table 1, the authors have aligned the 8 Standards of Mathematical Practices, a vital part of the Common Core State Standards (CCSS), with relative components of 3-Act Math Tasks. Even though some states have abandoned CCSS, many have retained the Standards of Mathematical Practice as a resource of importance within their state-wide mathematical academic standards. Table 2 aligns 3-Act Math Tasks with the National Council of Teachers of Mathematics' (NCTM) effective mathematics teaching practices (2020). The National Council of Teachers of Mathematics advocates that instructional programs from prekindergarten through grade 12 should enable each and every student to:

- Build new mathematical knowledge through problem solving.
- Solve problems that arise in mathematics and in other contexts.
- Apply and adapt a variety of appropriate strategies to solve problems.
- Monitor and reflect on the process of mathematical problem solving.

The Common Core State Standards describe the problem-solving process for mathematically proficient students in detail. In summary, students should be able to analyze the meaning of the problem, develop multiple pathways to a solution, then evaluate their own work for accuracy. The expectations are similar to those of NCTM.

CCSS Standards of Mathematical Practice	**Alignment with 3-Act Tasks**
Make sense of problems and persevere in solving them.	Students work through a process with little information given by the teacher.
Reason abstractly and quantitatively.	Students make sense of the numerical values that might exist in the 3-Act Math Task. To do this requires the playing out of various scenarios mentally before putting pen to paper.
Construct viable arguments and critique the reasoning of others.	Using this strategy, learners are asked to justify their methodology before they get to the final act. By partnering up with peers, students can compare their methodology with others, and troubleshoot with others.
Model with mathematics.	Students create a visual model or representation of the information they are given in Acts 1 and 2. Modeling is an important skill in Common Core, and in most states' math curriculum standards.
Use appropriate tools strategically.	Learners must make decisions about what mathematical tools are needed to find the solution.
Attend to precision.	Students will need to use equations, tools, and symbols correctly to arrive at a solution.
Look for and make use of structure.	By finding a pattern in the clues within the Acts, students make use of structure.
Look for and express regularity in repeated reasoning.	Learners make connections to prior learning and develop fluency by finding a shortcut while reworking the task in a different, more concise way.

***Table 1**. CCSS Standards Alignment*

NCTM Teaching Practices	**Alignment with 3-Act Math Tasks**
Establish mathematics goals to focus learning. Effective teaching of mathematics establishes clear goals for the mathematics that students are learning, situates goals within learning progressions, and uses the goals to guide instructional decisions.	3-Act Math provides an engaging context for the use of mathematics and the development of mathematical understanding.

Implement tasks that promote reasoning and problem solving. Effective teaching of mathematics engages students in solving and discussing tasks that promote mathematical reasoning and problem solving and allow multiple entry points and varied solution strategies.	Students have opportunities to practice estimation and reasonableness. Multiple approaches are encouraged.
Use and connect mathematical representations. Effective teaching of mathematics engages students in making connections among mathematical representations to deepen understanding of mathematics concepts and procedures and as tools for problems solving.	Presented with a visual prompt and little information, students are challenged to make connections between their own prior knowledge of mathematical strategies and how to best utilize that knowledge to arrive at a solution.
Facilitate meaning mathematical discourse. Effective teaching of mathematics facilitates discourse among students to build shared understanding of mathematical ideas by analyzing and comparing student approaches and arguments.	Students are curious and actively engaged as they wonder what will happen next. This approach provides opportunities to talk about mathematics and compare results.
Pose purposeful questions. Effective teaching of mathematics uses purposeful questions to assess and advance students' reasoning and sense making about important mathematical ideas and relationships.	This strategy creates situations which require students to engage in mathematical modeling. It helps students build relational understandings among mathematics concepts.
Build procedural fluency from conceptual understanding. Effective teaching of mathematics builds fluency with procedures on a foundation of conceptual understanding so that students, over time, become skillful in using procedures flexibly as they solve contextual and mathematical problems.	This strategy creates low barriers to entry, allowing the teacher to scaffold as necessary. Any level of learner can enter the task in Act 1. Students have opportunities to build new knowledge from prior knowledge.
Elicit and use evidence of student thinking. Effective teaching of mathematics uses evidence of student thinking to assess progress toward mathematical understanding and to adjust instruction continually in ways that support and extend learning.	3-Act Math Tasks provide opportunities for reflective thought. This approach asks students to share their thinking and justify their answers.

***Table 2**. NCTM Mathematics Teaching Practices Alignment*

Conclusion

3-Act Math Tasks are not just word problems, and they are not just ordinary math tasks. They promote higher-order thinking, strengthen problem-solving skills, and are widely accessible to diverse learners. We argue that the 3-Act Math Task strategy provides opportunities to tackle rich content using inquiry and a storytelling framework. Teachers can use this approach as a best practice to empower students to explore, persist, and collaborate. Each of the 3 steps is critical. Act 1 is the hook. It should be engaging and perplexing, spark curiosity, and create anticipation. A word of caution: these kinds of tasks take time, practice and patience; be sure you don't give your students all the information they need. Some of the best mathematical conversations happen when we ask our students to make predictions without having enough information! Act 2 is the meat of the task which promotes student thinking and solution seeking. Here we ask our students to wonder, to predict, and to guess. Act 3 is the climax and resolution of the dilemma. Using 3-Act Math Tasks, we can invite our students to try a task that is intuitive but lacking in detailed information, help them understand some math, and finally invite them to re-try the task only to discover that using math strategies is more efficient and more accurate. The benefits are enormous as students struggle to make sense of the problem, learn to generate questions to get more information, apply mathematical reasoning, and persevere. The goal is to make math contextual, challenging, and fun.

Lesson Title	Big Ideas	What do you wonder?
Peas in a Pod	counting	If all the peas were in one pod, how many peas would there be?
Dotty	counting and patterns	How many dots will be on the screen after the last bell?
the Candyman	counting and joining sets	How many candies are in are in his hand?
Share the Love	sharing quantities within 20	How many M&Ms will each girl get?
Counting Squares	counting and patterns	How many tiles are in the pile?
Stage 5 Series	counting and patterns	What will stage 5 look like?
Shark Bait	counting and joining sets through 20	How long is the worm?
Lil' Sister	comparing measurements	How much shorter is Lil' Sister than Big Sister?
Bag-O-Chips	building fluency through 10	How many bags of chips were missing?
Balancing Numbers	number combinations through	What is needed to make both side of the scale equal? (balance)
Humpty Dumpty	addition and subtraction within 20	How many eggs didn't break?
Popping Balloons	building fluency through 10	How many balloons are left?
the Cookie Monster	addition and subtraction within 50	How many cookies did the cookie monster eat?
the Pringle Ringle	addition and subtraction within 100	How many Pringles did it take to make the Pringle Ringle?

[the Juggler]	addition and subtraction	How many times will the juggler be able to juggle the ball until it hits the ground?
[Graham Cracker]	addition and subtraction within 100	How many crackers will fit inside the graham cracker box?
[Bright Idea]	addition and subtraction within 100	How many Skittles fit inside the light bulb?
[Snack Machine]	addition and subtraction within 100	How much did the Munchos cost?
[Sliced Up]	working with quarters and wholes	How many orange wedges are in the bowl?
[the Whopper Jar]	addition and subtraction within 100	How many Whoppers are inside the jar?
[It All Adds Up]	adding and subtracting money	What coins are in the bank?

Table 3*. Links to Recommended 3-Act Math Tasks (K-2)*

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Adventures with Dear Aunt Sally

by Bill Gillam

Abstract

The order of operations may not be as well-defined as we believe once implied operators and groupings are introduced. This article looks at discrepancies between various calculators, software, and texts in evaluating possibly ambiguous expressions.

Having an agreed-upon convention for what order to perform operations in arithmetic expressions helps ensure that two individuals who evaluate the same expression arrive at the same answer. This helps ensure that when a practitioner writes down a mathematical expression, other practitioners will have a clear understanding of the mathematical entity that is being communicated. Ideally, given an expression, there would be no ambiguity among practitioners on exactly how to evaluate it. That is not necessarily the case.

One popular mnemonic for remembering the order of operations for evaluating arithmetic expressions, PEMDAS, indicates recursively addressing items within Parentheses and other grouping symbols in the order of Exponents, then Multiplication and Division from left-to-right, and finally Addition and Subtraction from left-to-right. Most middle-schoolers are probably familiar with the phrase "Please Excuse My Dear Aunt Sally", with the suggestion that "My Dear" is a single salutation and "Aunt Sally" is a singular person to help remember the left-to-right precedence pairs.

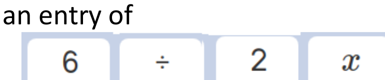
My experience with Aunt Sally and her ilk included an encounter with the order of operations as a computer science major and programmer. Different computer languages often have different mathematical syntax. "Three squared plus four" might look like "power(3, 2)+4", "3**4+4", "3^2+4", "(+ 4, (exp 3 2)) or something else depending on the language being used. The syntax for mathematical operations in a computer language is rigorously defined within that language, but the syntax may be slightly different in another language. Similarly, the rules for interpreting mathematical expressions by humans, while rigorously defined within a particular context, may vary slightly between different disciplines, practitioners, and publishing standards.

A few years ago, a group of educators from throughout South Carolina were discussing an expression similar to "6/2x." Given x = 3, most of them evaluated the expression as (6/2)*3, but a few claimed the juxtaposition in "2x" implies grouping, and the expression should be evaluated as 6/(2x). Both sides were able to justify their claim through specific examples of precedence. This group consisted of about a dozen teachers, leaders in their schools with a number of years of experience, most of who had earned master's degrees, a few had doctorates, and they could not positively agree on what the middle school level expression 6/2x meant.

Further investigation of this situation turned up a plethora of discussions and articles on this subject online. Another related situation that was encountered was that some calculators and testing software added unstated grouping rules to keyboard input to aid in editing for display on a screen, so that pressing a "/" key might have a different result than pressing a "÷" key.

For example, in one popular online program, the entry of

would result in a display of $\frac{6}{2x}$, but an entry of



would result in a display of 6÷2x. This results in a situation where the operators "/" and "÷" and can be interpreted differently between and even within various environments in terms of both text formatting and calculation.

Conventions in the Field

I am hesitant to state that a particular interpretation of an expression is wrong simply because it does not reflect what I was taught, especially if it is consistent and has historical precedence. Rather than try to logically debate which interpretation of some ambiguous notations is "correct", let's instead look at how these situations have been handled in practice by calculator manufacturers, software developers and textbook publishers. We'll look at three examples. We'll take a fairly deep dive into the first one and look at the second two more briefly.

Problem expressions example #1: 6 divided by 2 times 3, and 6 divided by 2 times x.

"Six divided by 2 times 3" is a modified version of a problem that has trended on social media over the last few years. If written 6/2*3, the situation is pretty straight forward once the PEMDAS or an equivalent set of rules is accepted, but rewrite it with implied multiplication and there are precedents for interpreting 6/2(3) as either (6÷2)*3 or 6÷(2*3). A constant juxtaposed with a variable as in 6 ÷ 2x is even more likely to be interpreted as implied grouping.

Let's look at how various calculators interpret these expressions:

TI – 73 interprets 6/2(3) as $\frac{6}{2}*3$ and 6/2x as $\frac{6}{2}*x$ with no implied grouping

Image 1. *TI-73 screen output (Texas Instruments)*

The **TI-84** and **Ti-Inspire** produced similar results. All TI products appear to evaluate this expression similarly.

HP Prime: when you enter 6, ÷, 2, * and 3 from the keypad, the calculator rewrites it as $\frac{6}{2*3}$. The division sign opens a denominator that does not close until ▶ is entered.

The user has to enter 6 ÷ 2 ▶ × 3 to get an evaluation of (6/2)*3. (The HP also has a Reverse Polish Notation mode, but we will ignore that.)

Image 2. *HP Prime screen output (Hewlett Packard*

Casio fx-991EX rewrites the equation 6÷2(3) with an implied grouping, providing a different answer from 6÷2*3 on the Casio.

6÷2*3 =9

6÷2(3) is redisplayed as 6÷(2(3)) = 1

6÷2x is similarly redisplayed as 6÷(2x) = 1

Image 3. *Casio fx-991EX screen output (Casio)*

The default calculator on my computer screen seems to agree with TI.

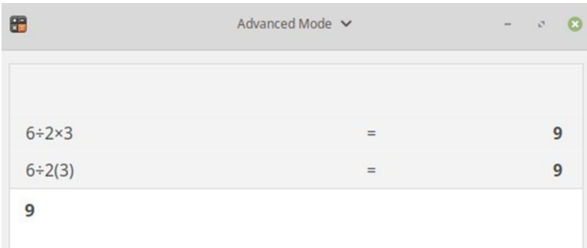


Image 4. *Linux Mint default calculator screen image (Burridge et. al.)*

There appears to be a mixed bag on calculators. Two of the calculators reformat the input to avoid the ambiguous form, the other two do not consider juxtaposition as a grouping notation.
Now lets look at some web-based calculating apps.

Desmos is a popular web-based graphing calculator. Desmos will display different forms of this equation depending on whether text is entered from the keyboard or cut and paste into it. If one types in 6/2∗3 directly from the keyboard, Desmos will interpret the slash as opening a division bar until manual closure and display this:

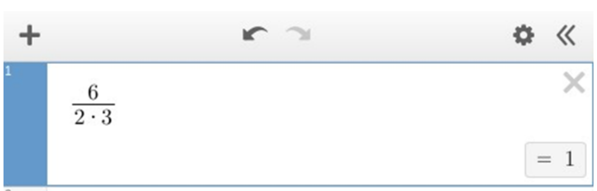


Image 5. *Desmos screenshot (Desmos)*

However, if I cut and paste any form of “6÷2*3” into the input area from another program, Desmos will not use the juxtaposition as grouping convention.

Image 6. *Desmos screenshot (Desmos)*

So the parser for Desmos will format the entry of 6/2*3 as a fraction with 2*3 as the denominator, but if you force the forms 6/2*3, 6/2(3), 6\2*3 or 6\2(3) by cutting and pasting, Desmos does not implement denominator grouping convention.

Mathway is a popular web-based calculating and tutoring program. Mathway has a “÷” and a “/” key input, and treats them differently. Mathway interprets “ /” as implying grouping, probably because they are treating it as a division bar from a formatting standpoint.

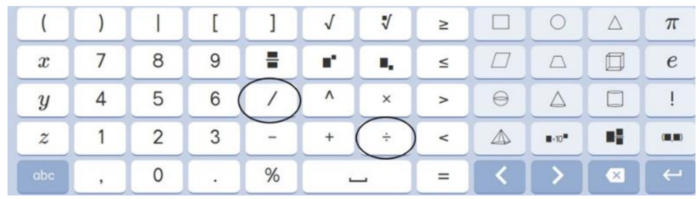


Image 7. *Mathway keyboard screenshot (MathWay)*

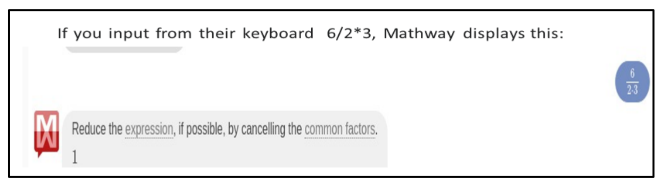


Image 8. *Mathway screenshot*

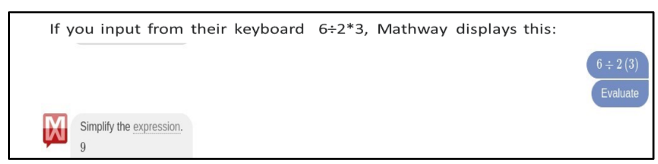


Image 9. *Mathway screenshot*

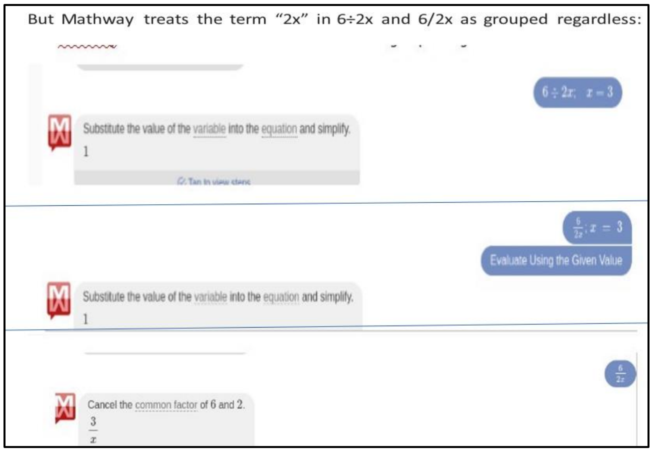


Image 10. *Mathway screenshots*

So for Mathway, "/", differs from "÷". The former opens a fraction bar, while the latter does not. In examples using the "÷" symbol, "2x" is grouped, but "2(3)" is not.

Wolfram Alpha is a popular web-based calculating app. Wolfram Alpha appears to never treat juxtaposition as implied grouping:

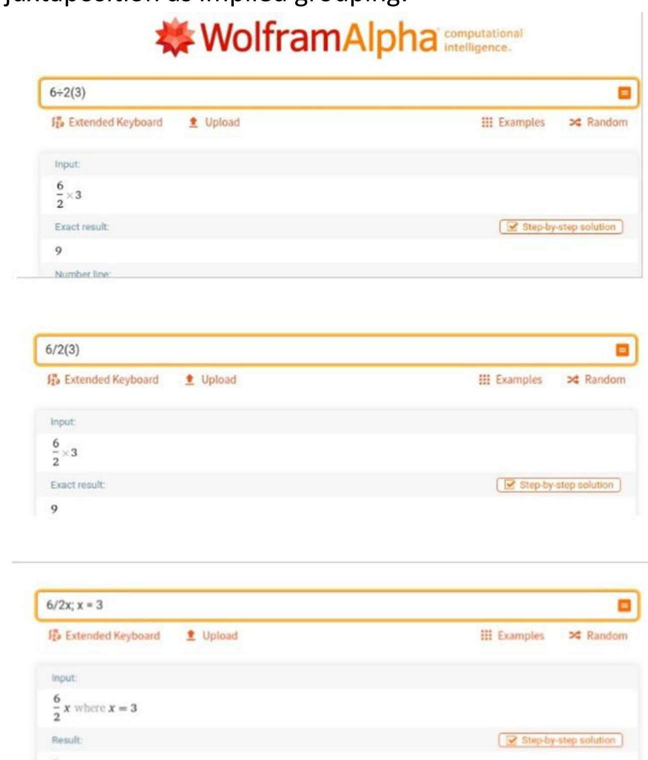


Image 11. *Wolfram screen shots*

Online apps also seem to be a mixed bag; many possessing their own peculiarities in terms of formatting and evaluation, except Wolfram which always seems to follow the PEMDAS with no implied grouping rule.

Spreadsheets

Most spreadsheets won't accept juxtaposition but will offer an alternate form that does not use implied grouping. Both LibreOffice and another popular spreadsheet program output something like this when attempted.

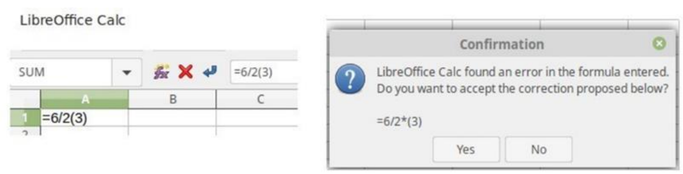


Image 12. *LibreOffice screenshot (The Document Foundation n.d.)*

Programming Languages

Java won't compile 6/2(3): (red underline indicates an error) or

6/2x:

6/2*x works:

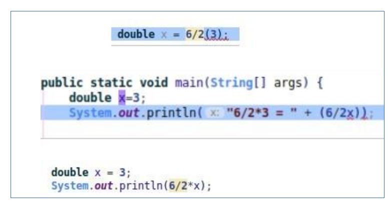


Image 13: *Java code example*

The "*" operator is even required in distribution.

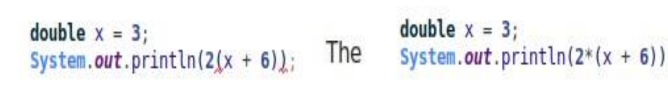


Image 14: *Java code example*

The syntax of java and most programming languages does not support juxtaposition as an operator, so the ambiguity is avoided.

Some Printed Examples

The College Algebra text (Miller, 2017) I currently teach from almost exclusively uses fraction/division bars and negative exponents rather than a binary operator like "/" or "÷" to indicate division, so the ambiguous situation is avoided.

The Abstract Algebra book I studied in college states: "We shall frequently delete the dot and write the product of *a* and *b* under ordinary multiplication as *ab*" (Burton, 1988, page 54). The phrase "ordinary multiplication" seems to indicate no implied grouping, however, this particular text exclusively uses fraction/division bars rather than a binary operator to indicate division.

The Feynman Lectures on Physics (Feynman, 2010) states a version of the Heisenberg Uncertainty Principal, $\Delta x\Delta px\Delta x\Delta pp\geq h/2$, as $\Delta x\Delta px\geq h/2\ \Delta x\Delta pp$ and a definition of Kinetic Energy, $K\ .E.=mV^2/2$ as $K\ .E.=WV^2/2g$ which are only correct if juxtaposition after the division sign is interpreted as grouping. Feynman appears to follow this alternative grouping rule throughout this text.

As a final example, a 1917 Article in the American Mathematical Monthly states that although the formal rules for evaluation indicate that $9a^2/3a = 3a^3$, the author stated: "But I have not been able to find a single instance where this is so interpreted. The fact is that the rule requiring the operations of multiplication and division to be carried out from left to right in *all cases* is not followed by *anyone*." (Lennes, 1917, page 94) While we will show some examples to demonstrate that multiplying and dividing from left to right in all cases has become more common in the current day, it looks like in 1917, a division sign usually indicated that everything juxtaposed to the right of a division was a divisor.

Unfortunately, these examples related to calculators, web-based apps and texts do not produce a clear preferred interpretation. It is difficult to say with absolute confidence what the "correct" interpretation of 6/2x is. A judicious use of parentheses or fraction bars can usually clarify this problem of multiple interpretations. The expressions 6/(3x),

$\frac{6}{3x}$ or $(6/3)x$ and $\frac{6}{3}x$

Depending on intent, are less likely forms to be misinterpreted.

Problem Expression Example 2: Negative Eight Squared

Do unaries take precedence over exponents? Most algebra text I am familiar with say no. Calculator output suggests no.

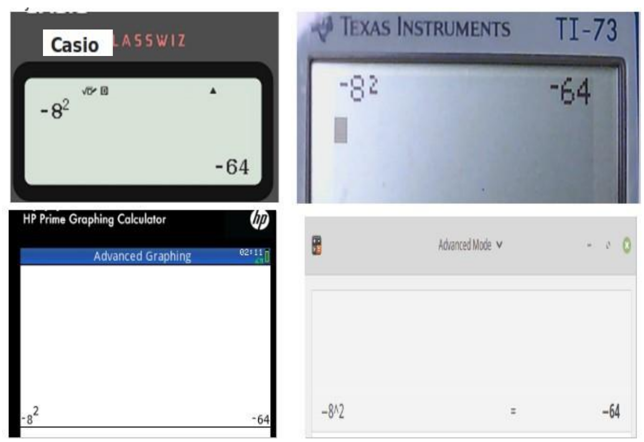


Image 15: *Calculator screens: Casio, Texas Instruments, Hewlett Packard, Burridge et al.*

Desmos, Wolfram, Mathway, and Java all also say no, but in both LibreOffice and Microsoft Excel, unary minus appears to take precedence over exponentiation.

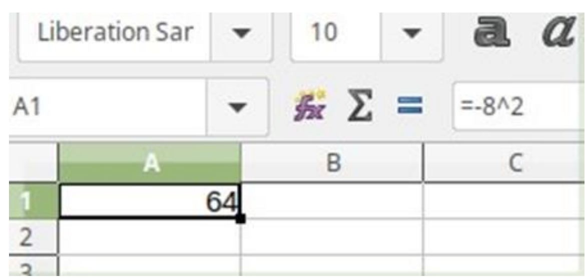
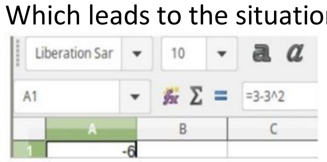


Image 16: *Spreadsheet Entry (The Document Foundation, n.d.)*

Which leads to the situation that $3\text{-}3^2\neq 3\text{+-}3^2$.



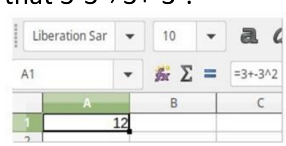


__Image 17:__ Spreadsheet Entry (The Document Foundation, n.d.)

Problem Expression Example 3: Four Cubed Squared

One generally accepted rule is:

$$a^{b^c} = a^{(b^c)}$$

Let's see how different calculators approach this.

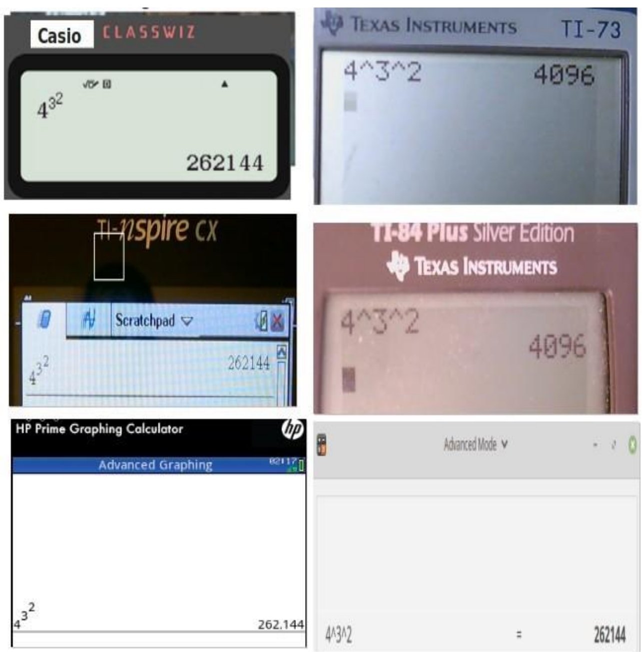


***Image 18.** Calculator screens: Casio, Texas Instruments, Hewlett Packard, Burridge*

TI isn't even consistent across its products line on this one. The others seem to follow the web-based applications Desmos and Wolfram use.

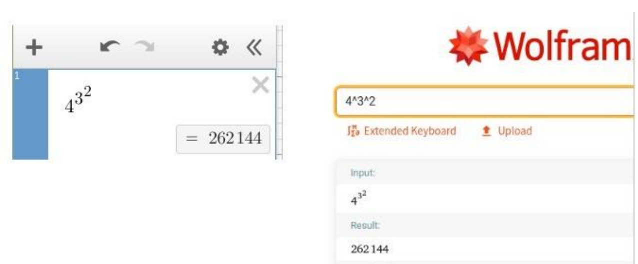


***Image 19.** Web applications screen shots from Desmos and Wolfram Alpha*

Spreadsheets do not:

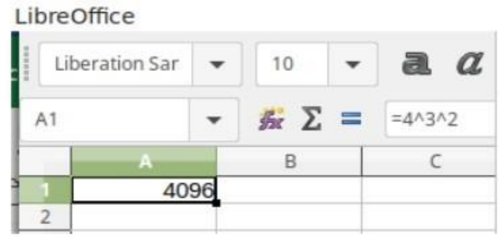


__Image 20:__ Libre Office Screenshot

Spreadsheets and some TI calculators do not follow the rule. As with the other cases of possible ambiguity, judicious use of parentheses can avoid multiple interpretations.

Summary

Most statements of order of operations mention grouping without specifically stating all allowed direct or implied grouping notations. This allows for some interpretation of what constitutes proper grouping notation. This can lead to expressions that can be legitimately interpreted in more than one way. Different texts, calculators and software packages may vary in these interpretations. How do we address this?

I recently had a student tell me that he was having a discussion with his wife about the evaluation of a problem they saw on social media, 6/2(2+1). He thought this evaluated to 9 following what was taught in our college algebra class software, notes and text. His wife believed the answer was 1. My response was that if his wife thought that the answer was 1 because she incorrectly believed multiplication was performed before division through a misinterpretation of PEMDAS, she was working the problem wrong. If she thought the answer was 1 because she interpreted juxtaposition as a grouping symbol, I could not say she was wrong, Casio and HP calculators would agree with her, but I could say that most modern math texts I was familiar with did not state or use the grouping rule she was using.

The examples provided were purposefully restricted to the basic operations of addition, subtraction, multiplication, division, exponentiation and unary negation. Even at that arithmetic operator level, My Dear Aunt Sally appears to be behaving differently, depending on whose house she's in. So if you are reading Feynman's lectures, realize that juxtaposition, particularly in a divisor, is going to imply grouping. If you are following the vast majority of modern math texts, it probably does not. Casio and TI calculators will treat juxtaposition differently, as do the various web-based apps. The grouping rules are typically consistent within any one environment but can differ between them. We need to make ourselves aware of differing house rules as we implement the tools of mathematical communication and practice and may want to avoid notations with multiple accepted interpretations if possible.

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Educator Perceptions: Effects of STEM Programs on Academic Success

by Marsha Neal

Abstract

The academic success of students is the key to education. Unfortunately, not all middle school students, grades 6-8, across southeastern South Carolina are performing at a level of academic readiness for the next grade level. The purpose of this qualitative case study was to explore perceptions of the effectiveness of STEM initiatives on student academic success at three middle schools in southeastern South Carolina.

STEM has been the basis for breakthroughs, technological innovations, and inventions throughout American history (National Science & Technology Council, 2018). STEM education is an interdisciplinary approach to learning which ties rigorous academic concepts with real world lessons (Bess, 2019). Students apply STEM in contexts that connect school, community, work, and global enterprise (Bess, 2019).

Believed to be vital in promoting innovation, efficiency, and general economic growth, STEM education is perceived as crucial throughout many countries as it is believed to be vital in promoting innovation, efficiency, and general economic growth (English, 2017). Mounting global attention has focused on STEM initiatives because these skills have been in higher demand, not only in STEM careers but in various other occupations (English, 2017).

Literature Review

When best practices are used in education, students reap the benefits (Bicer et al., 2015). Best practices for effective education included integrating mathematics and science for teaching STEM (Margot & Kettler, 2019). Teaching STEM in grades K—12 is vital to develop an interest in students in one of the STEM-related subjects (Bicer et al., 2015).

Through teaching integrated STEM in grades K—12, students are encouraged to succeed in school and consider a STEM-related career in the future (Kier & Blanchard, 2020). STEM schools are necessary for students, no matter where students live, the academic achievement or social background of the student (Bicer et al., 2015). A gap existed in literature as little research had been conducted on STEM education in middle schools, with most of the research being conducted in grades 9 — 12 (Blotnicky et al., 2018).

Research Questions

Research Question 1: What did middle school educators feel were the benefits students received from participating in STEM programs, based on the results of the standardized tests for the state?

Research Question 2: How did educators describe the experiences of students while participating in STEM initiatives?

Research Question 3: How did educators perceive strategies of STEM education to be effective in helping middle school students achieve academic success?

Data Collection

The data collected for the study captured the perceptions from three types of educators in middle schools that implemented STEM programs. Fifteen educators from three middle schools in three districts in southeastern South Carolina took part in the study. It is a common practice for teachers of all subject areas to collaborate during their planning sessions, therefore the participants had a mix of educational roles. One of the ELA teachers taught Reading 180, a program for students who were two or more grade levels below in reading. One special education teacher, one instructional coach, and two administrators were also part of the study.

Results

Six themes emerged from the data collected from the educators from the three middle schools across three school districts in southeastern South Carolina. The emerging themes were academic achievement, how

STEM fosters critical thinking skills, long-term impact on students, implementation strategies, challenges in implementation, and the needs of teachers to implement STEM initiatives. The themes which emerged answered the three research questions (see Table1).

Research Question	Themes
RQ1. What did middle school educators feel were the benefits students received from participating in STEM programs, based on the results of the standardized tests for the state?	Theme 1: Academic achievement
	Theme 2: Critical thinking
RQ2. How did middle school educators describe the experiences of students while participating in STEM initiatives?	Theme 2: Critical thinking
	Theme 3: Long-term impact
RQ3. How did middle school educators perceive strategies of STEM education to be effective in helping middle school students achieve academic success?	Theme 4: Implementation
	Theme 5: Challenges
	Theme 6: Teacher needs

***Table 1.** Research Questions and Themes*

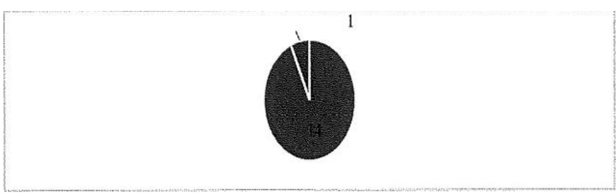

***Figure 1.** Participants who believe STEM benefits all students*

Participant perceptions of student academic achievement improved in many aspects, not only in math and science. Participant C3 taught the same students in eighth grade as taught in sixth grade. Looping means teachers teach the same students in different grade levels. Two years later, the looping gave Participant C3 a perspective on the changes over the two years

Participant C3 responded to the interview question, what are differences noticed pertaining to student perseverance in terms of STEM education? as follows:

> *Students who are in eighth grade now are able to use better and different strategies than the same students did in sixth grade. The students have a much greater strength in problem solving and 1<10w how to take apart questions, in order to find the answers. Improvement in academic achievement occurred across the curriculum.*

Social studies teachers perceived students gained a deeper understanding of history and knew how history compared to society today. Social studies teachers observed students relating geography to science, as with the study of migration, wetlands, and pollution. According to teacher observations, students become more proficient writers because of the cross-curricular concept of STEM. Educators perceived the improved academic achievement in students and the higher quality of lesson planning by teachers, which correlated to improved instruction in the classroom.

Theme 1: Academic Achievement. Teachers gave students real-world situational problems the teachers believed had no easy answer or solution. The teachers and instructional coaches perceived the situations challenged the students' abilities and allowed for problem solving. According to the participants interviewed, the ability to think critically allowed the students to find solutions to the problems at hand. In the questionnaire, Participant C2 responded,

> *Students develop problem-solving skills by practicing things like project-based learning, and by having an education which frames problems in a way which need to be solved by using skills obtained in multiple disciplines. This is a much more realistic approach for students, since in life do we rarely have problems which can be solved by just one discipline.*

Participant C3 answered questionnaire Question 4 pertaining to how students develop problem solving skills:

> *By collaborating with each other. They brainstorm ideas together, work with different opinions and ideas, they learn how to verbalize with each other productively, they practice/manipulate the problem at hand while making mistakes but figuring out by themselves how stuff works on their own.*

Theme 2: Fosters Critical Thinking. Critical thinking skills impacted students' academic achievement and were part of the experiences the students obtained through the STEM initiatives. Participants perceived the experiences obtained by students not only fostered the use of critical thinking skills during the activity but also taught the students new critical thinking strategies from peers during the activity.

Theme 3: Long-Term Impact on Students. A second insight within the theme of long-term impact for students was how STEM could bring a student out of poverty. Perceptions became apparent in the coding, indicating STEM allowed students to become active members of society while allowing a plentitude of opportunities not customarily be provided to the students.
In terms of long-term impact on students, Participant A2 commented,

> *Students can see different careers, what people do and how they use STEM. Students need to know problem-solving aspects for all jobs. The problem-solving skills help build on for the future and the students need to know different techniques.*

Theme 2, described under Research Question l, additionally tied into Research Question 2. Critical thinking skills impacted students' academic achievement and were part of the experiences the students obtained through the STEM initiatives.

Theme 4: Implementation Strategies. Regardless of if the school had a school-wide STEM initiative or a lab or pre-engineering class available for some students, the main implementation strategies focused on real-world applications, culminating projects, hands-on learning, and problem-solving. Of the 169 total usages of codes for implementation strategies,
90, or 53%, came from the four areas.

Participant A4 stated, "Consistent focus needs to be on application, with less rote knowledge. Application facilitates the process of more skills being learned, which are based on the interest of the students." As the case study was completed in middle schools in southeastern South Carolina, many participants perceived real-world field trips as a significant implementation strategy, living close to beach areas. Participant Bl described a yearly kayaking trip all students take, based around STEM:

> *[The kayak trip] allows for students to solve real-world problems outside of the classroom. During the yearly kayak trip, students not only test the runoff water for water quality and pesticides, but look for pollution based off fish signs, read and write about water quality, using the book "River of Words" and complete an art project after the trip. Participant A4 noted the school district offers a summer STEM camp, stating, "It is awesome, I even send my own kids to the camp."*

Theme 5: Challenges to Implementation. The data showed most educators who participated in the study agreed STEM initiatives benefitted all students. The participants realized initiatives needed to be implemented across the curriculum, but many spoke of the challenges in implementing STEM. To fully implement a program, all staff members needed to be on board, and in some instances, not all teachers were willing to participate.

A lack of funding and resources hindered implementation. Some grants were sought and awarded, but not always. Teachers felt burdened by the amount of time involved in implementing STEM initiatives fully. Participant C3 mentioned during the interview, "Time management is a huge challenge for teachers; there is never enough time to do all of what is wanted or required to do."

Sixteen percent of the responses regarding the challenges in implementation theme cited science, technology, engineering, and math being too time-consuming (see Table 7). The high teacher turnover negatively impacted implementation, as Participant A4 responded, "Parents and teachers do not always stay involved. At times, it is hard to maintain personnel who are willing to be in charge."

Theme 6: Teacher Needs to Implement STEM. Throughout the questionnaires and interviews was an overwhelming desire for the students to obtain the best education possible. However, 15 different responses by the participants revealed the teachers did not feel qualified to implement STEM initiatives. The participants noted teachers did not feel confident due to a lack of understanding of how to implement STEM initiatives. Teachers perceived college did not adequately prepare them in STEM implementation. Seventy-five percent of the teachers believed if more assistance, including professional development, were offered, implementation would be easier and more effective.

Conclusion

The qualitative case study provided several key findings. Fifteen educators from three middle schools in southeastern South Carolina took part in the case study. The participants had experience within a school which had a STEM initiative. Fourteen of the 15 participants believed STEM benefitted all students, especially in teaching students to use critical thinking and problem-solving skills to complete real-world, hands-on projects.

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Practicing math: strategies to support student motivation

by Nick Wallace

Abstract

Mathematics engagement across student populations can be difficult to depict and describe because of the multitude of factors that influence engagement. Motivation can be seen to form the center of the discussion on engagement and positive outcomes. Specifically, high mathematics motivation results in the greatest potential for learning, achievement, and further engagement. This article attempts to address two useful strategies to build both extrinsic and intrinsic student motivation and offers implications for educators.

Introduction

Educators in mathematics often face similar challenges concerning student discouragement and lack of engagement. While a myriad of factors may contribute to lack of engagement and motivation, it can hardly be argued that low student confidence levels and inadequate scaffolding and support hold insignificant roles. In contrast, student motivation, as driven by feedback, resiliency skills and internal aspirations, can be the determining factor that influences individual learning outcomes. The purpose of this research is to highlight the issues faced by mathematics educators, address possible solutions that address the needs of a diverse array of students and encourage more engagement in classroom-based learning.

Literature Review

Mathematics educators have long searched through a plethora of methods to increase student engagement, motivation, and involvement in the classroom. Morano et al. (2021) offer three strategies that include praise and a token economy, self-regulation, and the high-preference technique. Their research focuses heavily on both ways to increase student intrinsic and extrinsic motivation by encouraging students while also providing concrete rewards where appropriate.

First, they began by using external praise and acknowledgement of student efforts and successes. Offering verbal acknowledgment by educators and other facilitating individuals has been shown to offer moderate increases in student confidence, which contributes to intrinsic motivation. In support of this strategy, Begeny et al. (2020) conclude that reinforcement and feedback offer encouragement when also paired with group rewards. Reinforcement techniques encourage students' resiliency and problem-solving attitudes.

Their second focus on self-regulation techniques works in conjunction with their first by further increasing student persistence, self-recognition and resilience. Kim et al. (2015) describes this regulation as student-specific in which individual students learn to recognize challenges, address problems, and utilize skills to solve troublesome problems.

Thirdly, Morano et al. (2021) address the impact of the high-preference choice technique to build motivation and confidence. This technique presents students with a choice board of activities that they may complete in any desired order. The intended goal is to allow students to complete activities about which they hold the most confidence first, while also building confidence for more complex, longer, and higher-order tasks. Banda et al. (2009) underscore the ease with which the high preference strategy may be implemented by "identif[ing] students' high and low preferences to math tasks, develop[ing] and present[ing] simple worksheets during regular math instruction, and employ[ing] simple data collection methods to assess the effectiveness of the intervention (p. 149). This strategy holds great potential as a direct result of its wide applicability and low preparation requirements.

Implementation

Student motivation may hold the key to improving individual performance in, as well as willingness to engage in, mathematics tasks. As made evident, motivational strategies address both extrinsic motivation-related issues (rewards for target behavior) as well as intrinsic motivation-related issues (low self-confidence and persistence). Therefore, mathematics educators may increase student motivation by providing both feedback-based systems and preference-based activities.

Feedback and Rewards

Educators may begin by providing academic feedback through a token economy that relies heavily on academic feedback given directly to students. To adopt this approach, it is important to first detail the target behaviors, attitudes, perspectives, and skills that should be observed during the school year. In math, these may include varied problem-solving strategies, utilization of mathematics vocabulary, collaboration with classmates, and completion of homework and classwork assignments. Then, the educator can monitor for these behaviors, offer verbal reinforcement, and provide a token for this behavior. The intention of this practice is to encourage students to both recognize and recreate target behaviors by drawing their awareness to the class's actions. Additionally, token economies provide a reward that is tangible, material, and immediate to further reinforce behaviors.

Further reinforcement can also be provided in the form of academic feedback and acknowledgement of student effort. Often, students are unmotivated to engage in academic pursuits or attempt mathematics problems out of fear of failure as well as lack of confidence and support. Educators can meet this need by discussing student concerns, acknowledging positive instances of effort and problem-solving skills, and providing verbal encouragement. Then, reinforcement can be further enhanced by bridging the gap between extrinsic and intrinsic motivation. Students may be extrinsically motivated to engage in activities when they receive a token and/or verbal praise. To bridge the gap, educators can provide students with skills to self-regulate, monitor individual challenges, recognize efforts and successes, and produce further problem-solving techniques.

High Preference Models

While individual and class reinforcement can begin to build the necessary skills for self-regulation and resilience that are quintessential to sustained motivation, these skills can be further developed through promoting in-class assignment choice. Implementation of a preference-related class model begins by recognizing individual student preferences as well as aligning activities with both content requirements and state standards. After the preliminary step has been achieved, educators then create a choice board of unit-specific activities that must be accomplished within a particular time frame. Students are then presented with the choice board on which they can select activities in any order. This may be difficult to implement if the activities are aligned with lessons across a broad unit, and thus it may be most effective if implemented toward the end of a unit with well-defined tasks. For example, if students are required to multiply and divide decimals fluently as part of a unit on number sense, student choice boards may include utilizing the standard algorithm and long division techniques, calculating menu prices, playing a multiplication and division-based computer game, and solving related word problems. While students may not choose to address the word problems primarily, they may begin with the computer game. Activities such as this may provide and reinforce necessary skills of multiplication and division and promote accuracy, which builds students' ability levels and corresponding confidence. When students begin to address the standard algorithm and long division, they will have already reinforced their basic skills and thus be more able to achieve success. Finally, as they build to the higher order activities such as the menu and word problem scenarios, they will already possess the confidence and supportive skills that are required to engage in these activities well. High preference models can effectively scaffold and support student skill building toward higher order tasks when implemented with a variety of tasks.

Conclusion

Student motivation in mathematics education has proved to be an elusive metric. While some authors directly address and research student motivation, it can be difficult to observe the impact of individual activities and practices on student motivation and engagement. Primarily, student-specific interests and differences darken the picture surrounding the influences on student motivation. For example, students are all motivated by differing goals, interests, and aspirations, which are influenced by family and cultural values, past experiences and learning styles. For this reason, it can be difficult to engage in pedagogy, techniques and practices that reach the goal of improving student motivation and engagement in mathematics learning.

Researchers focus on select strategies to increase both extrinsic and intrinsic motivation. They focus primarily on increasing extrinsic motivation through praise (positive reinforcement) and a token economy. Token economies have been implemented with fidelity in various classrooms and environments and have been shown to offer an opportunity to encourage students using currency or points, which are given because of engagement in desired activities. This suggestion offers an introductory method toward raising student motivation that first aims to increase engagement. There are numerous scenarios in which the use of a token economy could prove useful, including building a classroom culture and improving educational environments, reinforcing uncommon but necessary behaviors, and teaching new skills.

Additionally, intrinsic motivation can be increased by teaching regulation strategies and employing the high preference technique. Both strategies allow students to focus on their struggles with the content by recognizing struggles, planning to complete practice problems, and building confidence for less preferred activities. The high preference technique offers promise to educators who experience a high level of difference between students in terms of motivation, drive and ability. This technique allows confidence building by presenting students with activities that can be more easily completed and thus require less support. Continually, it also allows for student choice, which helps to build autonomy and engagement. Therefore, the high preference technique should be utilized more heavily across classroom environments to increase student motivation in mathematics. This strategy offers direct connection to autonomy, agency and choice that allows students to engage with mathematics content in a relatable and attainable manner. Educators may benefit from employing the high preference strategy (and its simplistic design) when dealing with higher-order and deeper concepts.

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